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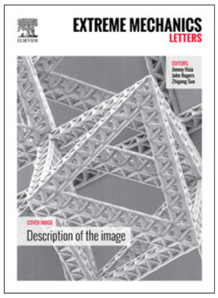

Flexel ecosystem: Simulating mechanical systems made from entities with arbitrarily complex mechanical responses

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Graphical Abstract

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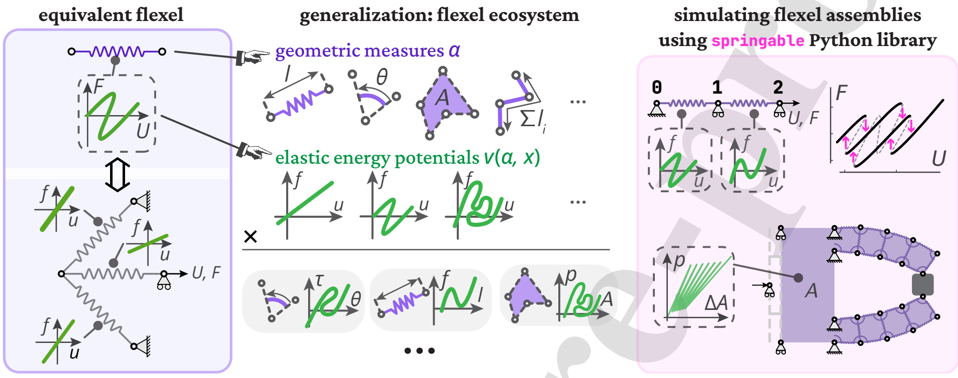

Highlights

Flexel ecosystem: simulating mechanical systems made from entities with arbitrarily complex mechanical responses

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- Flexels are energy-based elements featuring multivalued force-displacement curves.
- The flexel formulation yields an ecosystem of elements to build reduced order models.
- Flexels can represent nonlinear springs, flexures, cables, fluids or contact.
- The arclength method enables nonlinear static simulations of flexel assemblies.
- The framework is implemented in an open-source Python library.

Flexel ecosystem: simulating mechanical systems made from entities with arbitrarily complex mechanical responses

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Abstract

Nonlinearities and instabilities in mechanical structures have shown great promise for embedding advanced functionalities. However, simulating structures subject to nonlinearities can be challenging due to the complexity of their behavior, such as large shape changes, effect of pre-tension, negative stiffness and instabilities. While traditional finite element analysis is capable of simulating a specific nonlinear structure quantitatively, it can be costly and cumbersome to use due to the high number of degrees of freedom involved. We propose a framework to facilitate the exploration of highly nonlinear structures under quasistatic conditions. In our framework, models are simplified by introducing 'flexels', elements capable of intrinsically representing the complex mechanical responses of compound structures. By extending the concept of nonlinear springs, flexels can be characterized by multi-valued response curves, and model various mechanical deformations, interactions and stimuli, e.g., stretching, bending, contact, pneumatic actuation, and cable-driven actuation. We demonstrate that the versatility of the formulation allows to model and simulate, with just a few elements, complex mechanical systems such as pre-stressed tensegrities, tape spring mechanisms, interaction of buckled beams and pneumatic soft gripper actuated using a metafluid.

With the implementation of the framework in an easy-to-use Python library, we believe that the flexel formulation will provide a useful modeling approach for understanding and designing nonlinear mechanical structures.

Keywords: nonlinearity, instability, simulation, reduced order model, nonlinear spring

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1. Introduction

These last years have garnered a significant interest in understanding how to leverage large deformation, nonlinearities and instabilities to design structures capable of complex yet functional mechanical responses [1, 2]. For example, compliant mechanisms leverage flexibility to mimic conventional mechanism usually composed of many stiff components connected by joints [3]. Soft robots utilize shape changes to be intrinsically more robust, adaptable and autonomous, often by harnessing mechanical instabilities to embed sequenced [4], asymmetric [5], fast [6] or amplified [7] actuation. Recent progress in mechanical metamaterials have exploited large geometric changes and nonlinearities to achieve increasingly complex deformation pathways, enabling multistability [8] or high energy dissipation [9]. While the advancements driven by the exploitation of nonlinearities increase the capabilities of these fields, they also introduce new challenges for simulation and design.

Analysis of structures in the small-deformation regime is well established due to their linear behavior. However, large geometric changes introduce nonlinearities that make their study significantly more complex. Nonlinear finite element analyses suffer from limitations that make them less applicable for conceptual design, and do not always contribute to the understanding of systems that undergo large deformation and instabilities. For example, due to the high number of degrees of freedom involved, they can have prohibitive computational costs, fail upon facing mechanical instabilities or distorted meshes, and act as black boxes that are challenging to gain insight from.

To cope with these difficulties, reduced order models that are characterized by fewer degrees of freedom offer a more practical alternative to finite element analyses. They are easier to define, solve, and interpret by promoting qualitative understanding over quantitative accuracy. They usually employ fewer elements which together are still able to capture the qualitative phenomenology. For example, the pseudo-rigid body model uses a combination of rigid links and torsional springs to represent flexible, slender parts

[3]. Trusses of bars can be used to model and even inverse design multistable compliant structures [10]. Cosserat rods have shown great promise for modeling soft robotic [11] or soft living [12] systems. Simple spring or beam models that can be expressed analytically have been valuable in gaining insight into a wide variety of geometrically nonlinear mechanical structures [13, 14, 15, 16, 17, 18, 19].

Essentially, such existing approaches aim for a higher-level description of the mechanical system. Instead of describing the system as a mesh of many material elements, combining fewer abstract components allows to construct models that are cheaper to solve and easier to understand. However, strongly nonlinear mechanical behaviors such as nonmonotonic or multi-valued force-deformation paths are still either modeled from the bottom-up, by combining multiple more simple components, or by using overly abstract models such as 'hysterons', which require complex modeling to connect them to physical systems and moreover can lead to ill-defined systems [20, 21, 22, 23].

Here, we introduce an easy-to-use framework that defines components at a level of abstraction that allows them to single-handedly and intrinsically capture highly nonlinear static mechanical responses that are traditionally achieved using multiple components. Our formulation is based on energy potentials tuned to be stationary on prescribed, arbitrarily complex, possibly multi-valued generalized force-displacement curves. We show that this energy-based formulation can generate a broad ecosystem of mechanical components, with a wide range of geometries and intrinsic complexities. We also show that the framework allows for a relatively straightforward approach to build aggregate models where the various interactions between simple and complex elements can be simulated and explored. We conclude this work by demonstrating that our framework allows to model, simplify, and simulate a wide variety of mechanical systems studied previously that are subject to, e.g., geometric nonlinearities, pre-stress, hysteresis, buckling, snapping, contact, cable-driven or pneumatic actuation.

2. Overview

Before introducing the full ecosystem, let us stress that structures composed of linear springs can exhibit nonlinear force-displacement relations resulting from geometric nonlinearities. A classic example is the Von-Mises truss, which, despite being composed of three linear springs, produces a non-monotonic (Fig. 1a) or even a multi-valued force-displacement curve

(Fig. 1b), depending on the relative stiffness of the springs. The foundation of our approach is to represent such structures by single entities, which we call 'flexels'. As shown in Fig.1c,d, the intrinsic nonlinear behavior of a flexel can be tuned to capture, and be equivalent to, the geometric nonlinear behavior of compound structures, such as the Von-Mises trusses shown in Fig. 1a,b (Movie S1).

More formally, a flexel is defined as a deformable element whose elastic energy directly depends on its geometric measure α, a scalar quantity determined from the coordinates of the nodes composing the element, such as its length. The intrinsic nonlinear behavior of a flexel is defined by a generalized force-displacement curve, which relates the derivative of its elastic potential with respect to the geometric measure (that is, the generalized force f) to the change in geometric measure (that is, the generalized displacement $u = \Delta\alpha$). This relation encapsulates the nonlinear response and is used to construct an energy potential from which the ingredients needed for numerical simulations can be derived (SI, Section S1.1).

Flexels can be assembled to investigate how interactions between individual nonlinear elements give rise to more complex collective responses (SI, Section S1.2). For instance, coupling two flexels each characterized by a non-monotonic and multi-valued force-displacement curve in series reveals an equilibrium path with multiple turning points, indicating the presence of a snapping sequence upon loading and unloading (Fig. 1e, Movie S1). By assembling these flexels in other configurations, the interplay between intrinsic and geometric nonlinearities can be explored. For example, the same pair of flexels, assembled now at an angle and loaded from their connection point, produces a different snapping sequence, due to the additional geometric nonlinearities (Fig. 1f, Movie S1). The more complex force-displacement responses exhibited by the assemblies shown in Fig. 1e,f can in turn be replicated by a single flexel (Fig. 1g,h). Our formulation enables simulation at a higher level, eliminating the need of simulating the individual components of larger structures that flexels are intended to mimic. Still, it should be noted that while flexels provide a powerful approach to reduce the number of degrees of freedom while maintaining nonlinear behavior, the abstraction subsumes the deformation of the internal degrees of freedom into the flexel behavior, thereby hiding them from the surrounding and precluding direct coupling between internal nodes or non-actuated nodal loading direction with other flexels. For example, the deformation of the top node in Fig. 1b is absorbed within its flexel abstraction (Fig. 1d) and cannot be coupled to other flexels. Similarly, the deformation of the top node of the assembly shown in Fig. 1f along the horizontal direction cannot be coupled to other elements when using its flexel abstraction (Fig. 1h). Information about these internal deformations is however not lost; it could be retrieved by mapping the flexel state back to the assembly that it mimics.

Simulating assemblies of flexels under quasi-static loading involves solving a system of parametrized nonlinear equations, which our toolkit achieves by implementing the arclength method [24] (SI, Section S2.2). This numerical continuation scheme retrieves an entire succession of deformed states at equilibrium even if those form a path with turning points, allowing for simulations of structures subject to snapping instabilities at either constant force or displacement. Note that if the equilibrium path splits into multiple branches at a pitchfork bifurcation (the hallmark of buckling instabilities), only one branch will be continued by the arclength scheme. This limitation is inherent to the arclength scheme itself, as it computes the equilibria along a branch sequentially, while only following one branch at the same time. The branch continued by the scheme is chosen by imperfections. In the absence of imperfections, the branch continued is typically the one that corresponds to unstable symmetric configurations. Manually adding imperfections in symmetric systems is therefore recommended to follow the "buckled" branches. In addition, the arclength scheme is unable to retrieve equilibria disconnected from the initial path. Alternative methods could be implemented to cope with these two limitations [25, 26].

3. Formulation of the flexel ecosystem

The most basic flexel can be derived from the generalization of a nonlinear spring. For a nonlinear spring, the axial force is computed in two steps: the length is determined from the coordinates of its end nodes, then passed to an energy potential whose derivative yields the force. More general, a flexel extends this idea in two ways.

3.1. Geometric measure

First, the notion of length is broadened into a *geometric measure*, noted α, that we define as any scalar quantity computed from a list of nodes' coordinates:

$$\alpha = \alpha(\boldsymbol{z}), \tag{1}$$

where $\boldsymbol{z} = [z_0, z_1, z_2, ...]^\top$ is the list of coordinates (each z_i being the horizontal or vertical coordinate of a certain node in space). A geometric measure can represent a length, an angle, an area, the total length of a path or the distance between a point and a line for example (Fig. 2a, specific equations are available in SI, Section S3). The deformation of a flexel is described by its *generalized displacement* u, defined as the change of its geometric measure α with respect to its geometric measure at rest α_0:

$$u = \Delta\alpha = \alpha - \alpha_0. \tag{2}$$

The force associated to the generalized displacement u is the *generalized force* f.

3.2. Multivalued force-displacement curve and bivariate energy potentials

Second, the class of energy potentials is widened to also represent generalized force-displacement curves that are *multivalued* (Fig. 2b). Such curves cannot be captured by a univariate potential, $v(\alpha)$, as the generalized displacement alone is not enough information to determine the state. Instead, they can be represented by *bivariate energy potentials*, $v(\alpha, t)$, where t is an additional degree of freedom introduced to disambiguate the state. From a given generalized force-displacement curve (potentially multivalued) defined by the parametric equations

$$\begin{cases} u_{\text{equilibrium}} &= a(t) \\ f_{\text{equilibrium}} &= b(t), \end{cases} \tag{3}$$

where a and b are two continuous functions, the potential

$$v(\alpha, t) = \frac{1}{2}k(t)\left(\alpha - \alpha_0 - a(t)\right)^2 + b(t)(\alpha - \alpha_0 - a(t)) + \int_0^t b(\tilde{t})a'(\tilde{t})\mathrm{d}\tilde{t} \tag{4}$$

produces equilibria along the prescribed curve, where $k(t)$ is a function constructed from a and b (Eq. (S146)) to satisfy stability constraints (detailed in SI, Section S4.2). A proof that such a potential is stationary along the prescribed path is given in Section. S4.2.

The given generalized force-displacement curve defined by Eq. (3) can have an arbitrary number of turning points and intersections, allowing flexels to encode information about their loading history [21], or capture snapping or countersnapping phenomena [27], for example. This allows a single flexel to represent a complex mechanical system by only using two degrees of freedom. Due to stability considerations (SI, Section S4.2), the only restrictions are that the curves must remain continuous and must not form loops in which the tangent vector crosses the *vertical upward* direction, which prevents some structures from being abstracted into single flexels.

3.3. Ecosystem

Thanks to the decoupling between geometry and intrinsic behavior, we can generate a whole flexel ecosystem by independently defining geometric measures on the one hand and generalized force-displacement curves on the other. The energy e of a flexel is then defined by combining both quantities as follows:

$$e(\boldsymbol{q}) = v(\alpha(\boldsymbol{z}), t), \tag{5}$$

where $\boldsymbol{q} = [\boldsymbol{z}^\top, t]^\top$ is the array of coordinates coupled by the flexel. The force vector $\boldsymbol{f}$ and stiffness matrix $\boldsymbol{k}$ associated to the flexel are obtained from the energy e, by combining quantities deriving from geometry and intrinsic behavior (SI, Section S1.1):

$$\boldsymbol{f} := \frac{\partial e}{\partial \boldsymbol{q}} = \frac{\partial v}{\partial \alpha} \begin{bmatrix} \partial \alpha / \partial \boldsymbol{z} \\ 0 \end{bmatrix} + \frac{\partial v}{\partial t} \begin{bmatrix} \boldsymbol{0} \\ 1 \end{bmatrix}, \tag{6}$$

$$\begin{aligned} \boldsymbol{k} :=& \frac{\partial^2 e}{\partial \boldsymbol{q} \partial \boldsymbol{q}^\top} \\ =& \frac{\partial^2 v}{\partial \alpha^2} \begin{bmatrix} \frac{\partial \alpha}{\partial \boldsymbol{z}} \frac{\partial \alpha}{\partial \boldsymbol{z}^\top} & \boldsymbol{0} \\ \boldsymbol{0} & 0 \end{bmatrix} + \frac{\partial v}{\partial \alpha} \begin{bmatrix} \frac{\partial^2 \alpha}{\partial \boldsymbol{z} \partial \boldsymbol{z}^\top} & \boldsymbol{0} \\ \boldsymbol{0} & 0 \end{bmatrix} \\ &+ \frac{\partial^2 v}{\partial \alpha \partial t} \begin{bmatrix} \boldsymbol{0} & \frac{\partial \alpha}{\partial \boldsymbol{z}} \\ \frac{\partial \alpha}{\partial \boldsymbol{z}^\top} & 0 \end{bmatrix} + \frac{\partial^2 v}{\partial t^2} \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & 1 \end{bmatrix}, \end{aligned} \tag{7}$$

When multiple flexels are coupled in a structure, the vectors $\boldsymbol{f}$ and matrices $\boldsymbol{k}$ serve as building blocks to construct the force vector $\boldsymbol{F}$ and stiffness matrix $\boldsymbol{K}$ of the entire flexel assembly (SI, Section S1.2). Those global quantities $\boldsymbol{F}$ and $\boldsymbol{K}$ can then be used to find the equilibrium points of the assembly through the arclength continuation scheme [24] (SI, Section S2.2).

The algorithm finds the equilibria by computing the stationary points of the total potential energy Π [28] for various levels of external force $\tilde{\boldsymbol{F}}^{\text{ext}}$, i.e.

$$\Pi(\boldsymbol{Q}, \tilde{\boldsymbol{F}}^{\text{ext}}) := E(\boldsymbol{Q}) - \tilde{\boldsymbol{F}}^{\text{ext}} \cdot \Delta\tilde{\boldsymbol{Q}} \quad \text{is stationary} \tag{8}$$

$$\Leftrightarrow$$

$$\frac{\partial \Pi}{\partial \tilde{\boldsymbol{Q}}} = \boldsymbol{0} \tag{9}$$

where $\tilde{\boldsymbol{Q}}$ are the free coordinates among all the coordinates $\boldsymbol{Q}$, $E = \sum e$ is the elastic energy of all the flexels, and $\Delta\tilde{\boldsymbol{Q}}$ are the displacement caused by the external force $\tilde{\boldsymbol{F}}^{\text{ext}}$.

All together, this formulation allows to pair geometric measures to curves to produce flexels, which can ultimately be assembled and used in simulations to model complex mechanical entities (Fig. 2c, Movie S2). Let us give a few examples. Pairing an angle to a multi-valued curve gives a flexel that models a flexure capable of snapping at constant angular displacement (Fig. 3a). Combining an area to a softening curve yields a flexel that mimics the behavior of a compressible fluid and can be used to model pneumatic actuation (Fig. 3b). The total length of a polygonal chain coupled to a stiffening bilinear curve gives birth to a flexel that models a rope that is yet to be taut and can be used in cable-driven systems (Fig. 3c). Pairing the point-line distance to a curve yielding a nonzero repulsion force only under a certain threshold gives a flexel that models contact (Fig. 3d).

4. Workflow

In this section we illustrate how our toolkit can be integrated to the workflow to design a structure with complex deformation pathways (Fig. 4, Movie S3). To illustrate this, we consider two physical structures fabricated in silicone rubber (Smooth-On, Smooth-Sil 945) [27], shown in Fig. 4a. We set the goal of predicting the experimental tensile response of the system formed by coupling them in series.

First, a tensile test is carried out on each structure during which the force is measured while increasing and decreasing the extension. This reveals nonmonotonic force-displacement curves (Fig. 4b). Second, for each experimental test, a Bezier curve of degree 4 is fitted to the experimental data by adjusting the positions of control points (Fig. 4c-left). Fitting such a curve consists of finding the positions of control points, which form a control polygon that shapes the curve. Third, each Bezier curve is represented by a string of text listing the coordinates of the control points, which can then be used to define the generalized force-displacement curve of a longitudinal flexel, modeling each structure as a nonlinear spring (Fig. 4c-right). Fourth, we model the assembly of the two structures by connecting the two flexels in series (Fig. 4d-right).

In practical terms, this model is written in an input file that describes the node positions, the boundary conditions, the flexels, and the loading steps (Fig. 4d-left). Note that before loading node 2, the structure is preloaded by applying a load on node 1 to account for the weight of the assembly. More details on how to interpret or compose such input file are provided in SI, Section S6.

Finally, we find that the simulated force-displacement curve is complex and characterized by many turning points (Fig. 4e). We performed the physical experiment by connecting both structures in series and conducting a vertical tensile test, to validate the approach. Good agreement has been obtained both quantitatively and qualitatively (Fig. 4e). If multiple physical building blocks are available and characterized, this approach allows to build a catalog of achievable nonlinear behaviors that can be combined to quickly simulate assemblies. By scanning different nonlinear behavior combinations, the approach can help identify which blocks should be assembled within a certain network to get a specific desired response [27].

5. Use cases

We next demonstrate the versatility of the ecosystem to handle use cases with various types of complexity (Fig. 5, Movie S4). As a first example, we examine tensegrity trusses, and set the goal of understanding the effect of prestress on their mechanical responses (Fig. 5a). We model a tensegrity truss presented in a previous study [29], by using longitudinal flexels. We assigned linear behaviors to the inner flexels modeling the struts, and piecewise linear behaviors to the outer flexels modeling the cables so that they resist tension but not compression (SI, Section S7.4 for the full model description). Prestress is modeled by prescribing incompatible rest lengths to the flexels, so that the struts and cables are initially pre-compressed and pre-tensioned respectively (Fig. 5a-bottom). Under load, we observe that the stress-free truss behaves as a mechanism in the small deformation regime, i.e. with zero initial stiffness, while the pre-stressed truss is stabilized by acquiring a nonzero initial stiffness (Fig. 5a). This confirms that this truss is a pure tensegrity structure [29]. Beyond small deformations, our approach is able to capture the nonlinearities originating from the rotation of the longitudinal flexels, allowing to study the behavior of such structures under larger loads.

As a second example, we examine a mechanical system involving the deformation of tape springs (Fig. 5b). Those are stiff flexible strips with highly nonlinear behaviors under bending, similar to the system presented in a recent study [30]. The goal is to simulate the formation of the kink under a compressive load, and understand how the kink travels when loaded directly. We model the tape spring as a relatively stiff path flexel combined with an angular flexel with a multivalued torque-angular displacement curve that mimics the behavior measured in [30], to model the bending behavior (SI, Section S7.4 for the full model description). Symmetry is broken by a small load acting downwards on the vertex of the angular flexel in order to avoid bifurcation. By first subjecting the system to a compressive load, we show that the tape spring buckles and suddenly forms a kink, yielding a complex force-displacement curve resulting from the interplay between global buckling and the intrinsic bending behavior (Fig. 5b-top). After this first loading step, the left-most node is clamped and the bending point is directly loaded, which shows deformation with quasi-zero stiffness (Fig. 5b-bottom). This simulation reproduces the soft deformation mode used by tape spring appendages for gripping [30]. The current implementation only allows to load along nodal coordinates, but the framework can naturally be extended to directly actuate the rest geometric measure of a flexel, allowing for different loading modes, e.g., angular loading, to model the various actuation modes of the mechanism [30]. This example also illustrates how this framework allows to handle contact, buckling, snapping and multiple loading steps.

As a third example, we examine a highly nonlinear system subject to contact: two beams that can buckle and touch when bent (Fig. 5c). This system of "bumping beams" has been studied in previous work [31, 32]. We set the goal of simulating the complex behavior of such a system and understand the effect of the separating distance between the two beams on the mechanical response. We model each beam as two linear longitudinal flexels hinging through a linear angular flexel. The middle nodes are initialized slightly inwards, so that the beams individually prefer to bend toward each other. We model the contact interaction using a distance flexel that provides a high repulsion force only when the distance between the two middle nodes approaches zero (Fig. 5c-left). When subject to a global compressive load, we reproduce the effect of the separating distance on the global buckling direction of these pairs of bumping beams [31] (Fig. 5c-right). Unlike previous work studying similar systems using dynamic simulations [31, 32], our static approach allows to retrieve stable and unstable branches (Fig. 5c-right), giving insight into the various equilibrium configurations. We note that as the number of beams increases, our approach becomes costly as many stable and unstable configurations will have to be computed.

As a last example, we examine a pneumatic gripper actuated using a metafluid (i.e. medium composed of collapsable capsules surrounded by a fluid [33]), and sets the goal of simulating the resulting gripping force as the gripper is pressurized and actuated. The metafluid itself is modeled using a single multivalued area flexel, that single-handedly captures the highly nonlinear pressure-volume response of previously studied metafluids [33]. The area of the flexel is delimited by the outer nodes composing the gripper. The gripper is modeled by a set of linear longitudinal and angular flexels to model the stretching, compressive and bending stiffness of the walls of the hollow structure. The inner longitudinal flexels are assigned a stiffer behavior than the outer ones to generate bending. The gripping interaction is modeled as a distance flexel that produces a high repulsion force only when the distance between the inner end nodes of the gripper fingers goes below a certain threshold (i.e. the size of the object being grasped). The system is loaded using a force that tend to decrease the size of the area flexel, effectively modeling pressurization (SI, Section S7.4 for the full model description). The simulation reveals that the metafluid modulates the gripping force, allowing for delicate grasping as previously demonstrated experimentally in [33]. This example shows that, instead of modeling the metafluid by adding degrees of freedom for each capsules, the curve itself is directly assigned to an area flexel, drastically reducing the number of degrees of freedom. By contrast to the modeling approach shown before [33], adding extra capsules does not cost any additional degrees of freedom. The price to pay for that improvement is that the curve should be determined in advance, via a traditional modeling approach or via an experiment [33]. Once it is known, it can be used to model systems where that fluid is used at a much cheaper cost, while still capturing the complexity.

Together, these examples show the versatility of our approach. Importantly, each example only required a few lines of code to be simulated (SI, Section S7.4). Note that, in all these examples, the initial positions of the nodes and the flexels have been manually set, as this work focuses on reduced order models with relatively low number of nodes and elements.

6. Conclusion

We introduced the concept of flexels, entities able to single-handedly capture the strongly nonlinear responses of compound systems. This facilitates the simulation of various complex mechanical systems, with a focus on nonlinearities and instabilities. To promote accessibility and adoption, the formulation is implemented in an open-source and user-friendly Python library [35].

Key to developing this framework was the generalization of the concept of nonlinear spring along two independent axes, geometry and intrinsic mechanical behavior, which naturally generates a broad ecosystem of easy-to-combine elements. This ecosystem can be extended by defining different geometric measures or families of curves, which opens route to define custom flexels that are suitable to tackle specific mechanical problems, while still being compatible with the previously defined ones. Because our formulation is energy-based, it enables the use of well-established algorithms as well as the traditional stability analysis tools. Furthermore, even though we focused on static simulations in this work, the modularity of the implementation allows for the extension of the formulation in dynamic contexts [36, 37], where the module to compute the nonlinear elastic forces can be re-used.

We close by listing future challenges and perspectives. Even though each flexel can capture complexity, its response is only governed by a single geometric measure, which can be an oversimplification of reality. Being able to define a flexel with force-displacement curves along more than one dimension could help build more abstract and reduced models. This can in principle be done in the current formulation by defining energy potentials with additional dimensions. For example, flexels whose energy depends on two deformation measures could help construct 2d metamaterial models where each unit cell is a flexel [38]. Flexels with a force-displacement curve that can be tuned via an external parameter could help model the interplay between active materials and geometric nonlinearities [39]. We believe that embracing the flexel formulation will simplify simulations, help gain insight into a large variety of nonlinear problems and strengthen the use of nonlinearities and instabilities as design paradigm for compliant mechanism, soft robots, mechanical metamaterials and nonlinear structures.

Data availability

The 'flexel' ecosystem is available on PyPI: `https://pypi.org/project/springable/1.0.1/` (see SI, Section S5.1 for installation instructions), source code: `https://github.com/ducarme/springable/releases/tag/v1.0.1`.

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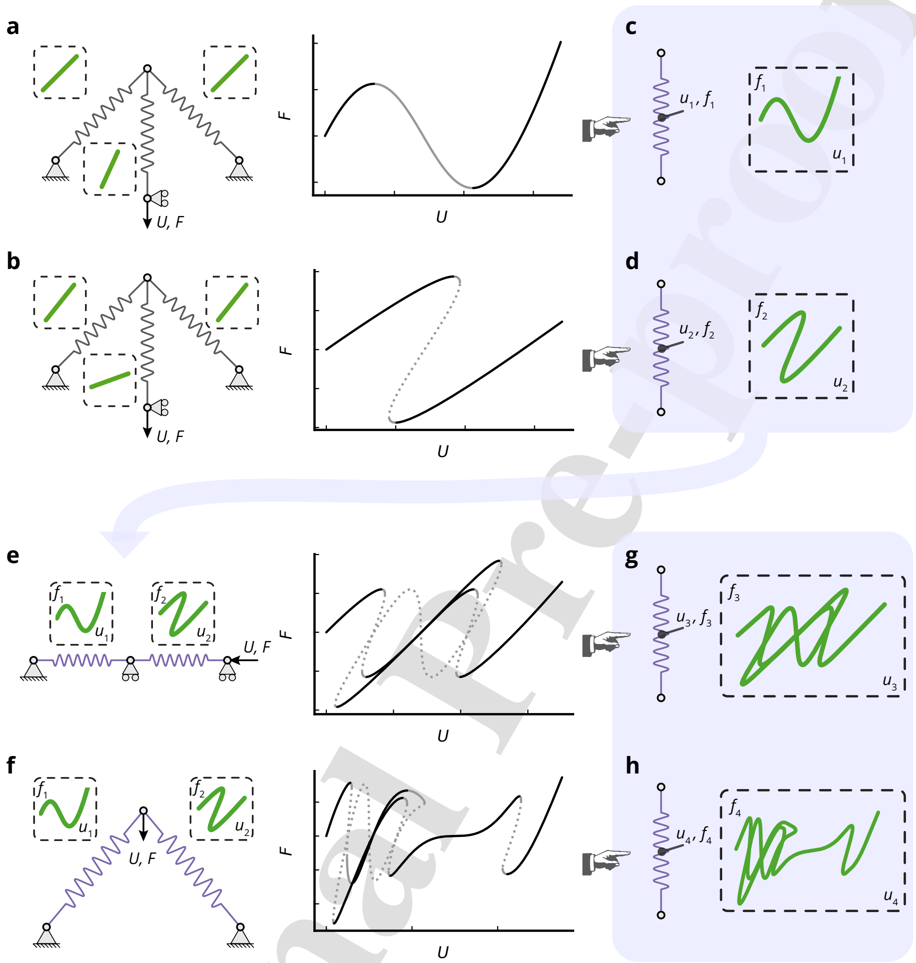


Figure 1: Construction of flexels. (a-b) Von-Mises truss composed of a pair of inclined linear springs (non-dimensional stiffness of 0.6 for (a), 1.0 for (b)) driven from their connection point through a third, vertical linear spring (stiffness of 20 for (a), 0.33 for (b)). The force-displacement response of the system is either (a) non-monotonic or (b) multi-valued. (c-d) Flexel equivalents of the Von-Mises trusses shown in (a) and (b), whose mechanical behavior has been tuned to mimic their force-displacement response. (e-f) Assemblies composed of nonmonotonic and multi-valued flexels as shown in (c) and (d) loaded in series (e) or at an angle (f), exhibiting complex force-displacement curves. (g-h) Flexel equivalents of the systems shown in (e) and (f). Black (gray) lines refer to states stable (unstable) under force-controlled conditions. Solid (dashed) lines refer to states stable (unstable) under displacement-driven conditions. The full descriptions of the models are provided in SI, Section S7.1.

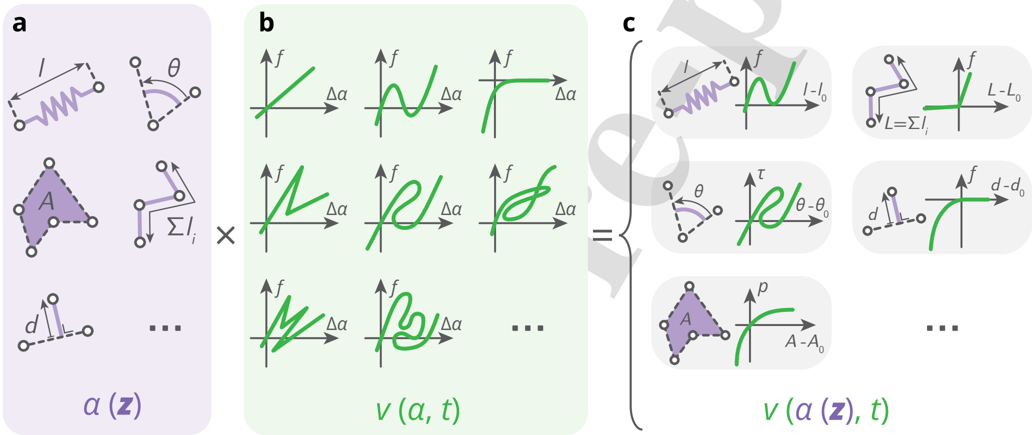


Figure 2: Flexel ecosystem. (a) Various geometric measures α computed from a list of node coordinates $\boldsymbol{z}$: length, angle, area, total length of a polygonal chain, distance point-line. (b) Various generalized force-displacement curves (linear, nonlinear, multi-valued) defining intrinsic mechanical behaviors via energy potentials $v(\alpha, t)$. (c) Examples of pairs of geometric measure and behavior forming the flexel ecosystem. The energy of a flexel is given by $v(\alpha(\boldsymbol{z}), t)$. More details on the definition of the geometric measures α and energy potentials v are provided in SI, Section S3 and S4.

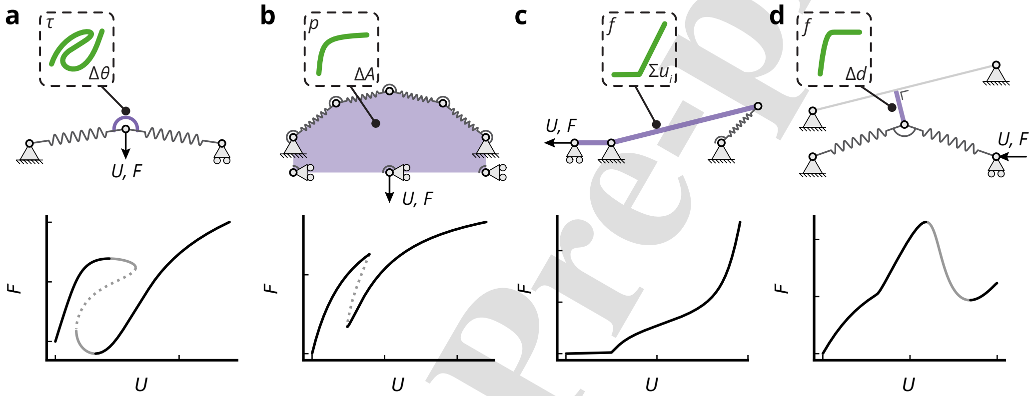


Figure 3: Examples of assemblies of flexels (top) that collectively produce force-displacement responses $F(U)$ (bottom). Black (gray) lines refer to states stable (unstable) under force-controlled conditions. Solid (dashed) lines refer to states stable (unstable) under displacement-driven conditions. Gray flexels are characterized by a linear generalized force-displacement curve. (a) An angular flexel with a multi-valued torque-angular displacement curve modeling a snapping flexure. (b) An area flexel with a softening pressure-areal displacement curve modeling pneumatic actuation. (c) A path flexel with a bilinear stiffening force-displacement curve, modeling cable-driven actuation. (d) A distance flexel with a force-displacement curve yielding a high nonzero force only for small distance values, modeling contact between a point and a line. The full descriptions of the models are provided in SI, Section S7.2.

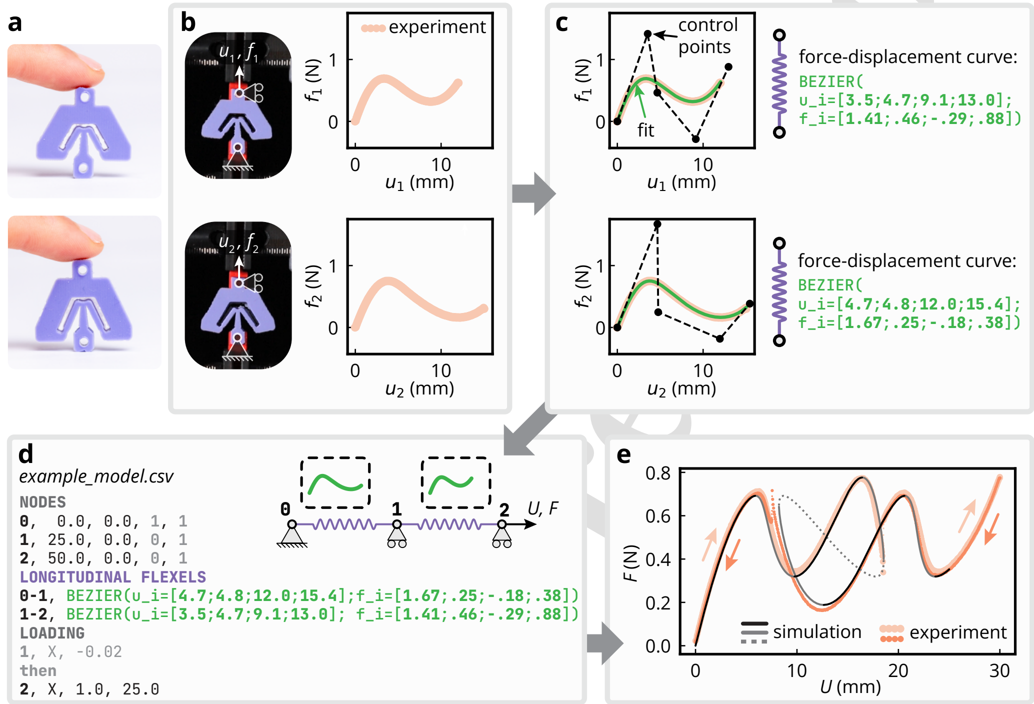


Figure 4: Workflow to simulate an experimental assembly of nonlinear building blocks [27]. (a) Two building blocks fabricated in silicone rubber. (b) Experimental force-displacement curves obtained by performing a tensile test on the building blocks. (c) Left: Bezier curves (solid green lines) fitting the experimental force-displacement curves (orange lines). The control polygons and the control points are depicted by the black dashed lines and the black dots. Right: Equivalent flexels with tensile behaviors defined by Bezier curves, specified by a string of text listing the coordinates of the control points (green text). (d) Flexel model describing the assembly of the serially-coupled building blocks (Right) and the input text file representing the model (Left). (e) Force-displacement curve of the serially-coupled assembly. Experimental data obtained during the loading (unloading) phase is depicted by light (bright) orange dots. Simulated data is depicted by black and gray curves, where black (gray) refers to states stable (unstable) under force-controlled conditions, and solid (dashed) curves refer to states stable (unstable) under displacement-driven conditions. Force and displacement are measured with respect to the preloaded configuration. The full descriptions of the models are provided in SI, Section S7.3.

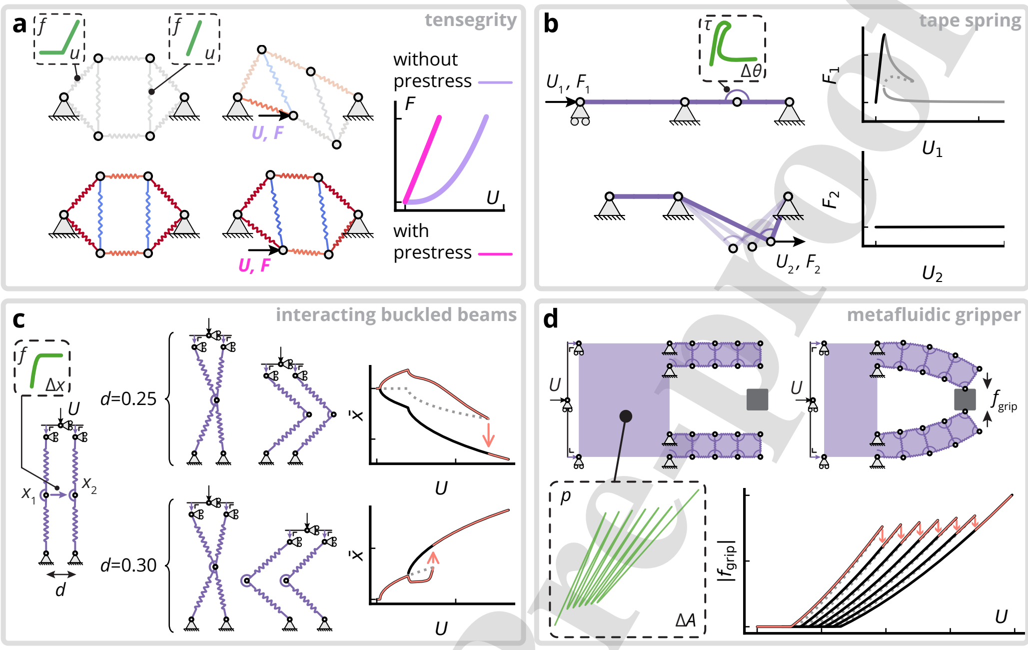
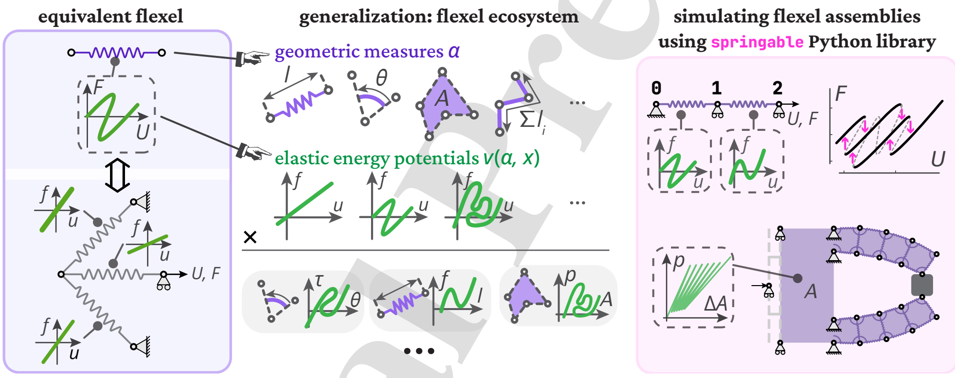


Figure 5: Examples of use cases. (a) Left: Tensegrity structures without prestress (top) and with prestress (bottom) subject to an external loading. Outer flexels model the cables through a piecewise linear behavior, whereas inner flexels model the struts. Flexels in compression, at rest and in tension are colored in blue, gray and red respectively. Right: Force-displacement curves of the tensegrity structures with and without prestress. (b) Left: Model of a tape-spring gripper [30] loaded in two steps. The first one applies compression to the tape spring, eventually triggering buckling and creating a kink (top). The second deforms the buckled tape spring by driving the kink (bottom). Right: Force-displacement curves corresponding to the first (top) and second (bottom) loadsteps. (c) Left: model of a pair of buckling beams separated by a distance d, loaded in compression and interacting through the contact of their middle node [34, 32, 31]. Center: deformation sequence of the beams when $d = 0.25$ (top) and $d = 0.30$ (bottom). Right: Deformation paths when $d = 0.25$ (top) and $d = 0.30$ (bottom). $\bar{x} := (x_1 + x_2)/2$. (d) Top: Model of a gripper actuated using a metafluid [33] to grasp an object. Bottom: Intrinsic behavior of the area flexel modeling the metafluid (left) and deformation path of the system (right), shown as the gripping force f_{grip} as a function of the applied displacement U. The full descriptions of the models are provided in SI, Section S7.4.

Highlights for article

"Flexel ecosystem: simulating mechanical systems made from entities with arbitrarily complex mechanical responses"

- Flexels are energy-based elements featuring multivalued force-displacement curves.
- The flexel formulation yields an ecosystem of elements to build reduced order models.
- Flexels can represent nonlinear springs, flexures, cables, fluids or contact.
- The arclength method enables nonlinear static simulations of flexel assemblies.
- The framework is implemented in an open-source Python library.



equivalent flexel
generalization: flexel ecosystem
simulating flexel assemblies
using springable Python library
geometric measures α
elastic energy potentials V(α, x)

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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