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VRIJE UNIVERSITEIT

ENGINEERING ILLUMINATION THROUGH MULTIMODE FIBERS FOR IMAGING AND METROLOGY

ACADEMISCH PROEFSCHRIFT

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aan de Vrije Universiteit Amsterdam,
op gezag van de rector magnificus
prof.dr. J.J.G. Geurts,
volgens besluit van de decaan
van de Faculteit der Bètawetenschappen
in het openbaar te verdedigen
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Engineering Illumination Through Multimode Fibers for Imaging and Metrology

Aleksandra Igorevna Ivanina

Ph.D. thesis, Vrije Universiteit Amsterdam, 2026

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“What we observe is not nature itself, but nature exposed to our method of questioning.” -
Werner Heisenberg, *Physics and Philosophy* (1958)

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1

INTRODUCTION

From the earliest moments of civilization, light has shaped human progress. It has illuminated the night, marked the passage of time, and inspired creation myths across cultures. As our understanding deepened, light became more than a practical necessity. It became a fundamental means of investigating both the natural world and the universe beyond.

The scientific study of light began in antiquity, when thinkers such as Euclid and Ptolemy proposed the first theories of vision [1]. In the medieval Islamic world, Ibn al-Haytham's Book of Optics introduced systematic experimentation, laying the foundation of modern optics [2]. Centuries later, Newton's prism revealed the composite nature of white light, while Huygens and Young demonstrated its wave-like behavior [3–5]. These early insights set the stage for the major theoretical advances of the nineteenth and twentieth centuries: Maxwell's theory of electromagnetism, which established light as an electromagnetic wave, and Einstein's concept of photons, which explained phenomena such as the photoelectric effect and ushered in quantum physics [6]. Together, they revealed the dual wave-particle nature of light that underpins modern science.

Hand in hand with theory came instruments that extended the reach of human vision. The microscope allowed us to see the microscopic world, and the telescope enabled us to gaze into the vastness of space [7, 8]. Through these instruments, people learned to use light as a tool, first to reveal what was hidden, and later to probe, manipulate, and measure the world with increasing precision. From oil lamps to electric bulbs, every advance in light sources expanded what could be seen and measured.

The first laser was demonstrated by Theodore Maiman in 1960, marking the beginning of coherent, controllable light sources and leading to the Nobel-recognized development of lasers [9, 10]. Lasers quickly transformed both technology and science: they became the backbone of global communications, the drivers of precision manufacturing, and the enablers of microelectronics. In particular, laser-based nanolithography revolutionized modern computing. Using light to etch circuits on a nanometer scale has made it possible to create the microchips that power today's phones, computers, and data centers. This ability to manipulate light at the smallest possible scales illustrates a broader theme of illumination, using light not only to observe the world around us, but also to build the technological foundations of the information age. The same mastery of light underpins progress across the

full spectrum of modern optical instruments. For example, in microscopy, confocal imaging enhances contrast and optical sectioning, whereas super-resolution techniques overcome the diffraction limit [11]. In astronomy, adaptive optics and space-based telescopes apply precise control of illumination to peer deeper into the universe, correcting distortions caused by Earth's atmosphere and capturing light from the most distant galaxies [12].

These developments demonstrate that progress in science and technology relies as much on the way we illuminate our subjects as it does on the methods we use to detect them. Despite these advances, there are still many challenges to overcome. Light can scatter, distort, or absorb in the materials we want to examine. Fragile structures are hidden, and complex environments resist illumination. At the same time, the features we seek to detect are becoming ever smaller, pushing the limits of precision. Thus, the history of optics—from ancient theories of vision to modern EUV-nanolithography machines—is not only a record of remarkable successes, but also a continuing story of challenges and open questions.

1.1 SPATIAL WAVEFRONT SHAPING

The idea of controlling the wavefront of light has its roots in astronomy. In 1953, Babcock proposed the concept of adaptive optics: by measuring and correcting distortions introduced by Earth's turbulent atmosphere, telescopes could, in principle, recover high resolution [13]. This vision was realized decades later with the development of fast wavefront sensors and deformable mirrors, allowing ground-based observatories to overcome "seeing" and produce images rivaling those from space [12].

In parallel, other approaches to undoing wavefront distortions emerged. Speckle interferometry demonstrated that the fine structure hidden by turbulence could be recovered by analyzing seemingly random *speckle patterns* [14]. When a coherent light beam passes through a rough surface or a medium that scatters light, many partial waves interfere to form a granular intensity distribution of bright and dark spots, known as a speckle pattern. Although speckles appear random, they are fully deterministic: the same input field produces the same speckle field as long as the medium remains unchanged [15]. Returning to the history timeline, optical phase conjugation (OPC), introduced in the 1970s, offered another route by generating a time-reversed copy of the distorted field that retraces its scattering path [16].

The breakthrough into strongly *scattering media* came in 2007, when Vellekoop and Mosk showed that coherent light could be refocused through an opaque layer of white paint by actively shaping the incident wavefront with a spatial light modulator (SLM) [17]. This experiment marked the birth of modern wavefront shaping (WFS) in complex media, turning scattering from a nuisance into a tool for imaging and control. Following the pioneering experiment, WFS rapidly developed into a vibrant research field. Follow-up work formalizes iterative focusing through turbid media [18]. In parallel, digital optical phase conjugation (DOPC) combined interferometric field measurement with SLM playback, providing a high-speed, camera-based implementation of time reversal [19, 20]. Together, these methods showed that light undergoing multiple scattering can be measured, predicted, and actively controlled. This capability has led directly to biomedical applications, where WFS has been used to refocus light deep inside living tissue, enabling high-resolution imaging and optogenetic stimulation beyond the ballistic regime [21–23]. Achieving such control experimentally requires instruments capable of independently modulating the phase

and amplitude of light with high precision.

Two main technologies dominate experimental work: spatial SLMs and digital micromirror devices (DMDs) [24–27]. Each comes with its own strengths and limitations, and both have been adapted with special methods to be able to control the phase and amplitude of the wavefront. Liquid-crystal SLMs are pixelated electro-optic devices that impose a continuously tunable phase delay on an incident optical field via electric-field-induced reorientation of birefringent liquid-crystal molecules. In contemporary WFS systems, the dominant architecture is the liquid-crystal-on-silicon (LCoS) SLM, operating in reflective phase-only mode. Here, the local change in refractive index modifies the optical path length of each pixel, yielding a programmable phase profile while preserving a nearly constant amplitude. This phase-only operation enables high diffraction efficiencies exceeding 80–90%, a defining advantage for high-fidelity beam shaping [27].

Although SLMs are primarily used as phase modulators, amplitude control is also possible, either by combining them with polarizers or by using complex encoding strategies [28]. Modern commercial LC-SLMs now feature megapixel resolutions, pixel pitches below 4 μm , fill factors above 90%, and phase quantization up to 12–16 bit, enabling exceptionally accurate wavefront control [26]. However, their principal limitation remains the slow electro-optic response of viscous liquid-crystal layers. Standard devices operate at video rates ($\approx 60 - 120$ Hz), while specialized overdriven or thermally accelerated variants reach only 0.4–1.7 kHz, with ferroelectric LC-SLMs exceeding 3 kHz only at the expense of binary phase modulation and poor efficiency [27]. Consequently, LC-SLMs are ideally suited for static or slowly varying scattering systems, but become suboptimal when the decorrelation time of the medium falls very quickly. By contrast, DMDs are MEMS-based reflective SLMs composed of arrays of tilting micro-mirrors, each switching between two stable angular states to direct light either into or away from the optical path. This enables binary amplitude modulation at refresh rates exceeding 20–30 kHz, orders of magnitude faster than LC-SLMs. Such ultrafast switching makes DMDs exceptionally well-suited for dynamic WFS in rapidly decorrelating media, including biological tissue, MMFs, and turbulent flows. However, DMDs are intrinsically amplitude-only devices. Another experimentally important advantage of DMD-based WFS is the ability to display truly static binary patterns without temporal flicker [29]. Unlike LC-SLMs, which require periodic voltage inversion to avoid ionic charge buildup and thus exhibit residual intensity oscillations, DMD mirrors remain mechanically latched in their state without refresh-induced modulation. This property is particularly advantageous in feedback-based WFS, where long integration times may be required to improve signal-to-noise, and temporal intensity fluctuations directly degrade phase estimation accuracy.

To achieve phase and complex modulation, indirect holographic encoding techniques are required, most notably Lee holography and the superpixel method [24, 30, 31]. In Lee holography, the desired phase profile $\phi(x, y)$ is embedded into a high-frequency binary diffraction grating. The encoded pattern generates multiple diffraction orders upon reflection, and a spatial filter placed in the Fourier plane selects a single first diffraction order carrying the target phase information. While this enables effective phase control using a binary device, the modulation remains quantized and approximate, with fundamental trade-offs between spatial resolution, phase depth, and diffraction efficiency.

Amplitude control can be added through duty-cycle modulation of the grating within

each superpixel: increasing the fraction of mirrors in the “on” state raises the intensity of the selected diffraction order, allowing independent tuning of amplitude and phase [32]. Although this approach is robust and fully compatible with the ultrafast refresh rates of DMDs, it is inherently inefficient for coherent beam control, with typically a few percent (1-10%) of incident power contributing to the desired diffraction order, and a drastic reduction of the effective degrees of freedom available for WFS [27, 31]. Very recently, a new class of MEMS-based phase-only light modulators (PLMs) has been demonstrated, bridging the performance gap between LC-SLMs and DMDs. These devices consist of megapixel arrays of micro-mirrors whose vertical piston displacement, rather than tilt angle, is controlled electrostatically, enabling direct phase modulation with 4-bit (16-level) precision. Unlike LC-SLMs, PLMs are polarization-agnostic, and unlike DMDs, they provide native phase modulation with high optical efficiency ($\approx 70 - 90\%$) [27].

In summary, complementary strengths make both devices, SLM and DMD, indispensable for WFS, depending on speed, which is the dominant requirement.

Beyond devices, the success of WFS relies on optimization algorithms. Feedback-based WFS algorithms rely on the principle that the intensity at a chosen target position provides sufficient information to improve the input phase pattern iteratively. The modulator is divided into segments, each representing one controllable degree of freedom. Physically, each segment of a DMD does not act as an ideal continuous phase shifter, but rather as a local diffractive grating element due to the finite micromirror size and periodic pixel structure. Under coherent illumination, the DMD therefore behaves as a blazed reflective diffraction grating, distributing optical power into multiple diffraction orders. By an appropriate choice of illumination angle (blazing condition), one diffraction order can be preferentially selected and treated as the effective output field used for feedback optimization, while the remaining orders are spatially filtered out. By varying the phase of these segments and recording the resulting feedback, the algorithm gradually aligns the contributions of all segments constructively at the target [29]. Different variants of feedback-based methods have been developed [18]. The continuous sequential algorithm updates each segment immediately upon measurement, the partitioning algorithm modulates large groups of segments simultaneously to increase signal-to-noise, and evolutionary or genetic approaches explore random trial fields to maximize the target signal.

The most common approach is the sequential stepwise algorithm. Phase-only modulation is achieved by dividing DMD into N independent segments, where N denotes the total number of controllable degrees of freedom. Each segment is optimized one after another, indexed by $k = 1, 2, \dots, N$ (Fig. 1.1(a)), while all other segments remain fixed and provide a static reference field at the target [18].

This reference is characterized by an intensity I_0 and a phase ϕ_0 , such that the corresponding complex field can be written as

$$E_0 = \sqrt{I_0} e^{i\phi_0}. \quad (1.1)$$

The contribution of segment k to the field at the target is described by an intensity I_{seg} and the phase ϕ_{seg} . Applying a controllable phase shift ϕ to this segment results in the field

$$E_{\text{seg}}(\phi) = \sqrt{I_{\text{seg}}} e^{i(\phi_{\text{seg}} + \phi)}. \quad (1.2)$$

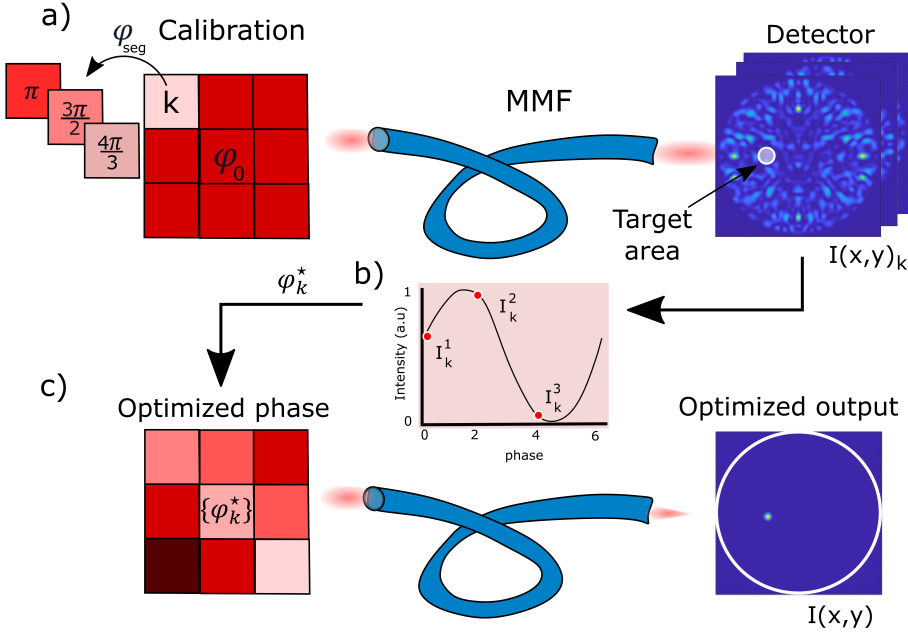


Figure 1.1: Sequential step-wise WFS algorithm (a) Calibration step: the DMD is divided into N independent segments. For the k -th segment, three phase values ϕ_{seg} ($0, 2\pi/3, 4\pi/3$) are projected while all other segments remain fixed with ϕ_0 , and the resulting target intensities $I(x, y)_k$ are recorded at the detector. (b) From the cosine dependence, the optimal phase ϕ_k^* for each segment is determined. (c) Measurement step: once all N segments are assigned with optimized phase $\{\phi_k^*\}$ is applied simultaneously. The contributions of all segments interfere constructively at the target, producing a focus at the targeted area.

The total field at the target is therefore

$$E(\phi) = E_0 + E_{\text{seg}}(\phi), \quad (1.3)$$

The detected intensity is given by

$$I(\phi) = |E(\phi)|^2 = E(\phi)E^*(\phi), \quad (1.4)$$

where $E^*(\phi)$ denotes the complex conjugate of $E(\phi)$. Substituting $E(\phi) = E_0 + E_{\text{seg}}(\phi)$ yields

$$I(\phi) = |E_0|^2 + |E_{\text{seg}}|^2 + E_0 E_{\text{seg}}^*(\phi) + E_0^* E_{\text{seg}}(\phi). \quad (1.5)$$

Using the definitions of E_0 and $E_{\text{seg}}(\phi)$ and Euler's formula $e^{i\theta} = \cos \theta + i \sin \theta$, the interference terms can be written as

$$E_0 E_{\text{seg}}^*(\phi) + E_0^* E_{\text{seg}}(\phi) = 2\sqrt{I_0 I_{\text{seg}}} \cos(\phi - \Delta\phi), \quad (1.6)$$

which leads directly to the cosine-law dependence of the detected intensity. Substituting the expressions above gives:

$$I(\phi) = I_0 + I_{\text{seg}} + 2\sqrt{I_0 I_{\text{seg}}} \cos(\phi - \Delta\phi), \quad (1.7)$$

where $\Delta\phi = \phi_0 - \phi_{\text{seg}}$ is the relative phase offset between the background field and the contribution of segment k . The intensity is maximized for $\phi = \Delta\phi$, which defines the optimal phase ϕ_k^* for that segment (Fig. 1.1(b)). As Eq. (1.7) shows, the detected signal exhibits a cosine dependence on the applied phase ϕ . Constructive interference is obtained when the phase of the segment matches that of the background field, $\phi = \Delta\phi$, which defines the optimal phase ϕ_k^* . To determine $\Delta\phi$ using only a small number of measurements, the intensity is recorded for a discrete set of known phase shifts. In this work, a three-step phase-stepping method is employed, in which the phases $\phi = 0, 2\pi/3$, and $4\pi/3$ are applied sequentially. The corresponding measured intensities are denoted by $I(0)$, $I(2\pi/3)$, and $I(4\pi/3)$, respectively. These measurements are combined into the complex quantity

$$S = I(0) + I(2\pi/3) e^{i2\pi/3} + I(4\pi/3) e^{i4\pi/3}, \quad (1.8)$$

from which the relative phase offset is obtained as $\Delta\phi = \arg(S)$. The optimal phase for segment k is therefore given by $\phi_k^* = \arg(S)$. In diffraction terms, the algorithm achieves wavefront control by phase-aligning local grating elements in a binary hologram, where grating position and duty cycle determine the phase and amplitude of the first diffraction order. After all N segments are calibrated, their optimal phases $\{\phi_k^*\}$ are applied simultaneously to the DMD. The interference of all contributions is now constructive at the target, and a bright diffraction-limited focus emerges at the output of the scattering media (Fig. 1.1(c)).

The performance of an optimization procedure is commonly quantified by the *enhancement*, defined as the ratio between the intensity at the target after optimization I_{opt} and the average intensity obtained for an unshaped (random) I_{rand} incident wavefront [17]:

$$\eta = \frac{\langle I_{\text{opt}} \rangle}{\langle I_{\text{rand}} \rangle}. \quad (1.9)$$

Physically, the enhancement reflects the degree to which the phases of multiple scattered field contributions are aligned at the target position. The enhancement therefore provides a direct measure of how effectively the available degrees of freedom are used to control light propagation through a scattering medium. In practice, the enhancement is further limited by experimental noise, finite modulation depth, and temporal decorrelation of the scattering medium. While the enhancement η measures the relative intensity increase at the optimized focus, it does not quantify how much of the total optical power is concentrated there. To assess the efficiency of power delivery, we therefore introduce the *power ratio* (PR), defined as the fraction of the total detected optical power contained within the optimized focal region:

$$\text{PR} = \frac{I_{\text{focus}}}{I_{\text{total}}}. \quad (1.10)$$

Here, I_{focus} denotes the integrated intensity within the target and I_{total} is the total transmitted intensity summed over the entire detection region. The achievable PR is constrained by the available degrees of freedom used to control the optical field. When only phase modulation is employed, the redistribution of optical power is limited. Full complex-field and polarization control allow more efficient energy redistribution and higher achievable power ratios [18, 33, 34].

1.2 TRANSMISSION MATRIX

In addition to WFS optimization algorithms, control of light propagation through scattering media can be achieved using the *transmission matrix* (TM) formalism. The TM describes the linear relationship between a set of input modes and the resulting output field. Once measured, the TM allows direct prediction and control of the transmitted field for any chosen input [33, 35–39].

For monochromatic light at fixed polarization, we represent the input field as a complex vector $\mathbf{a} \in \mathbb{C}^N$ and the output as $\mathbf{b} \in \mathbb{C}^M$, where N and M denote the number of controlled input and measured output channels. The TM $T \in \mathbb{C}^{M \times N}$ acts as a deterministic linear operator that transforms the input into the output:

$$\mathbf{b} = T \mathbf{a}, \quad (1.11)$$

with each column of T corresponding to the output field generated by a single input basis function. The choice of basis is flexible and may correspond to plane waves, Hadamard patterns, or pixel-based representations [40].

In practice, the TM of a scattering medium is commonly measured in the spatial basis defined by the experimental degrees of control and detection. Here, the input channels n correspond to discrete spatial segments on an SLM or DMD, while the output channels m correspond to pixels on the camera. The light propagation can then be written explicitly as

$$E_m^{\text{out}} = \sum_{n=1}^N T_{mn} E_n^{\text{in}}, \quad (1.12)$$

where E_n^{in} denotes the complex field modulated by the n -th input segment, E_m^{out} is the field measured at the m -th output channel, and T_{mn} are the complex-valued elements of the TM [35, 37, 41].

The number of required probe measurements is determined by the number of optical degrees of freedom in the illuminated region of the medium, which scales approximately with the area and numerical aperture relative to the wavelength. Each probe illumination fills one column of the TM, and by scanning through a complete basis of N inputs, the full mapping is assembled.

Each element T_{mn} encodes both the magnitude of transmission and the optical phase shift between input and output channels. Since cameras measure only intensity, dedicated strategies are required to recover this phase information [42, 43]. Interferometric approaches (off-axis holography, phase stepping) provide accurate and fast access to both amplitude and phase [35, 37, 44–49], but require stable interferometers. Non-interferometric or computational methods, such as OPC, Gerchberg-Saxton algorithms, phase diversity, and more recently, deep-learning approaches, avoid references at the cost of additional computation [42, 43, 50–52].

1.3 RANDOM ILLUMINATION FOR COMPRESSIVE IMAGING

Precise control of the illumination is not always required to extract useful information from a scattering medium. When coherent light propagates through a disordered system, random interference between multiple scattering paths gives rise to speckle patterns. Although these patterns appear random, they are deterministic and form a diverse set of illumination fields.

Rather than attempting to eliminate speckle, these random fields can be used directly as structured illumination. In particular, speckle-based illumination provides the multiplexed measurements required for *compressive sensing* (CS), enabling image reconstruction through computational inversion. CS is a particularly powerful technique. It enables accurate reconstruction from far fewer measurements than conventional sampling theory would require by leveraging the fact that many natural images are sparse or compressible in a suitable basis [53, 54]. Each structured illumination pattern generates a multiplexed measurement that mixes spatial information, and a collection of such measurements contains enough information to recover the image with significantly reduced acquisition time. Figure 1.2 illustrates this principle: in conventional raster scanning, each pixel must be measured sequentially, causing the acquisition time to grow linearly with image size, whereas CS captures the scene in a reduced set of structured measurements that jointly encode the full image content.

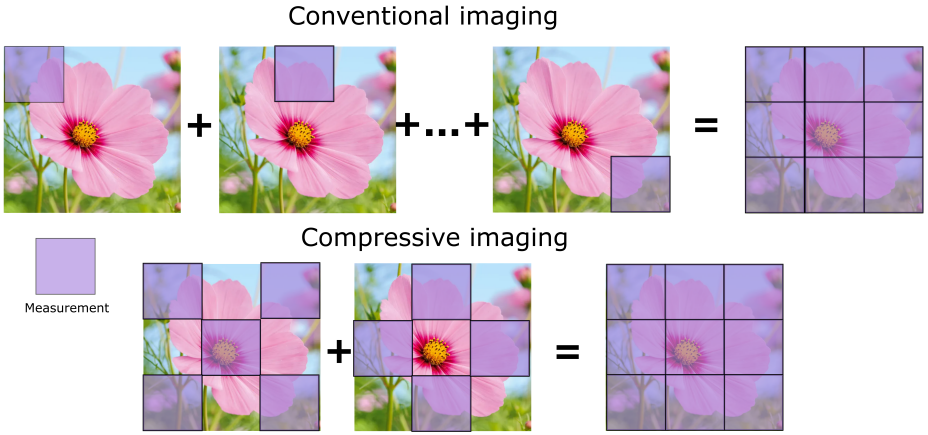


Figure 1.2: In conventional imaging (raster scanning), each pixel is measured individually, while CS reconstructs the object from far fewer multiplexed measurements.

According to the Nyquist–Shannon sampling theorem, perfect reconstruction of a band-limited signal is guaranteed only if the sampling rate is at least twice the maximum frequency [55]. For example, if audio contains frequencies up to 20 kHz, a digital recorder must sample at ≥ 40 kHz, which explains the 44.1 kHz standard for audio CDs. In imaging, the highest spatial frequency corresponds to the smallest resolvable feature, set by the optical cutoff

$$\nu_{\text{cutoff}} = \frac{2 \text{NA}}{\lambda}, \quad (1.13)$$

where λ is the wavelength and NA the numerical aperture. Thus, the sampling pitch must satisfy

$$\Delta x \leq \frac{1}{2\nu_{\text{cutoff}}} \approx \frac{\lambda}{4 \text{NA}}. \quad (1.14)$$

This condition ensures diffraction-limited resolution, but it also means that conventional raster scanning often collects far more data points than strictly necessary, limiting imaging

speed [56]. While the Nyquist–Shannon theorem applies to uniform sampling of arbitrary signals, natural images are rarely arbitrary. Most contain strong redundancies: edges, repetitive textures, smooth regions, or dominant low spatial frequencies. As a consequence, the underlying information can often be represented far more compactly than suggested by uniform sampling. Formally, let $x \in \mathbb{R}^N$ denote an image. Assume that x admits a sparse representation in a known transform basis $\Psi \in \mathbb{R}^{N \times N}$, such that

$$x = \Psi s, \quad (1.15)$$

where $s \in \mathbb{R}^N$ is the corresponding coefficient vector. If only $K \ll N$ of these coefficients carry significant information, the image is said to be K -sparse in the basis Ψ . The *sparsity fraction* is defined as

$$\beta = \frac{K}{N}, \quad (1.16)$$

where N is the total number of coefficients in the chosen basis (e.g., the number of pixels or Fourier coefficients) and K is the number of significant, non-negligible coefficients that actually carry information. Thus, β quantifies the fraction of essential coefficients: the smaller β , the stronger the sparsity and the greater the potential gains in compressive sensing [54, 57]. Even when signals are not exactly sparse, many natural images are *compressible*, meaning that most coefficients decay rapidly in magnitude and only a small fraction carries most of the information. This practical form of sparsity underlies the theoretical guarantees of CS and enables faithful reconstruction from far fewer measurements than required by conventional Nyquist sampling [54, 57]. Figure 1.3 illustrates this principle for the Fourier basis: most energy is concentrated in low spatial frequencies, and retaining only the largest K coefficients yields a visually faithful reconstruction.

Mathematically, the measurement process can be expressed as

$$y = Ax + n, \quad (1.17)$$

where $y \in \mathbb{R}^M$ is the vector of the detector signals, $A \in \mathbb{R}^{M \times N}$ is the measurement matrix formed by the recorded speckle patterns, $x \in \mathbb{R}^N$ is the unknown object, and n is measurement noise. Since typically $M \ll N$, this system is underdetermined. To make reconstruction possible, one assumes that x is sparse in Ψ , i.e. $x = \Psi s$ with only $K \ll N$ nonzero coefficients in s . Reconstruction can then be formulated as an ℓ_1 -minimization problem:

$$\min_s \|s\|_1 \quad \text{subject to} \quad \|A\Psi s - y\|_2 \leq \varepsilon, \quad (1.18)$$

where ε bounds the noise level. The recovered signal is finally obtained as $x = \Psi s$.

A wide range of reconstruction algorithms has been developed to solve the underdetermined inverse problem of CS. Classical approaches include ℓ_1 -minimization [53, 54], greedy algorithms such as orthogonal matching pursuit (OMP) [58], and basis pursuit [59]. For imaging applications, regularization strategies such as total variation (TV) minimization have proven particularly effective in preserving edges and fine structures [57, 60]. Other works explored Bayesian inference [61], approximate message passing (AMP) [62], and deep-learning-based reconstructions [50, 63], which offer dramatic gains in speed and robustness. Using sparsity and randomness, CS enables high-quality image recovery from far

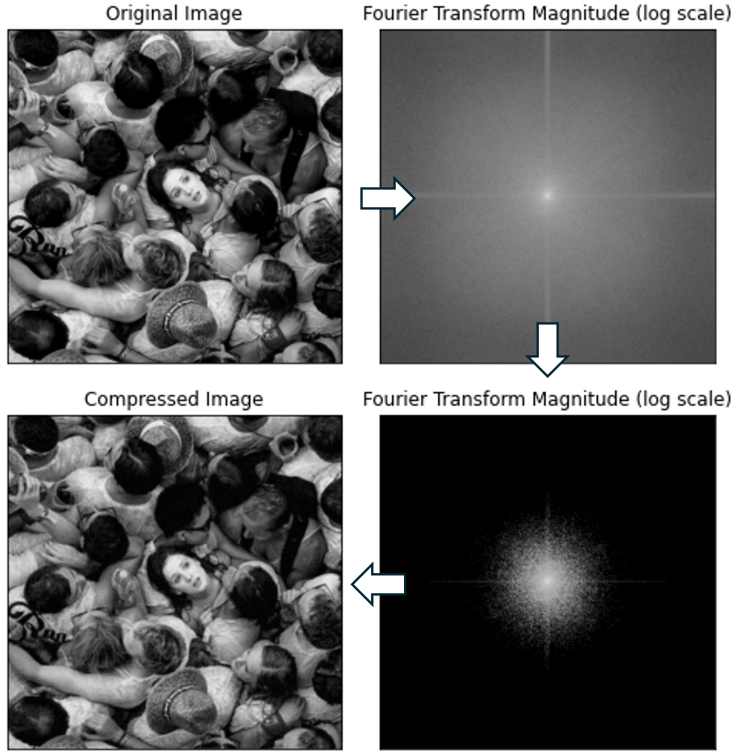


Figure 1.3: Sparsity in the Fourier basis. Top: original grayscale image (left) and its 2D discrete Fourier transform (right, log scale). Bottom: a sparse reconstruction obtained by retaining only the $K \ll N$ largest Fourier coefficients (Photo courtesy of David Ramos).

fewer measurements than traditionally thought possible, reducing acquisition times, pushing resolution beyond classical limits, and expanding the reach of optical imaging into regimes that were previously inaccessible.

1.4 OUTLINE

Illumination is a crucial factor in imaging: it can be manipulated, adjusted, and deliberately engineered to enhance image quality and extract more information. This thesis takes that perspective and uses optical fibers as a platform to show how illumination becomes a controllable design parameter rather than a fixed constraint for light delivery. Optical fibers are among the most versatile tools in modern photonics. Widely used for telecommunications, they guide light over long distances with low loss and minimal footprint. Because they are thin, flexible, and compatible with diverse optical sources, fibers have become important in many applications such as sensing, microscopy, and metrology. Inspired by this idea, the subsequent chapters address specific illumination challenges that we can solve using multimode fiber (MMF):

- **Chapter 2:** presents the theoretical framework for light propagation in step-index MMFs. It describes the formation of speckle patterns at the fiber output and analyzes their statistical properties, providing the foundation for understanding how MMFs generate complex illumination fields and how we can control them.
- **Chapter 3:** focuses on experimental ways for measuring the complex-valued TM of MMF, with particular emphasis on phase retrieval methods: in-line and off-axis holography. Both TM acquisition schemes are compared, and their performance is evaluated in terms of robustness, speed, stability, and ability to project simple and complex-shaped targets, highlighting how the choice of method depends on the target application.
- **Chapter 4:** demonstrates high-resolution compressive sensing through a flexible (single-mode-MMF) probe using wavelength-dependent speckle illumination. Image reconstruction is achieved with a narrow spectral bandwidth (< 10 nm), while maintaining strong temporal stability and robustness against single-mode fiber bending. The approach combines computational and experimental methods to enable fast and flexible sample reconstruction.
- **Chapter 5:** proposes and experimentally demonstrates the first implementation of complex speckle illumination for quantitative phase imaging, with the speckle patterns generated using an MMF. The phase information of pure-phase objects is retrieved through a non-interferometric and non-iterative reconstruction method, enabling diffraction-limited phase reconstruction.

Together, these chapters demonstrate that optical fibers are not only passive waveguides but active components of the imaging system. By combining physical insight, wavefront control, and computational strategies, the thesis shows how illumination through an MMF can be treated as a designed and engineered parameter.

2

2

LIGHT PROPAGATION THROUGH A MULTIMODE FIBER

Multimode fibers are the primary optical elements studied in this work. They support the propagation of multiple spatial modes, which interfere at the output and produce complex speckle patterns. While this modal interference can be seen as a drawback, the large number of supported modes provides a high-dimensional optical space that can be exploited for controlled illumination. This chapter describes the basic properties of step-index multimode fibers, the generation and characterization of speckle fields.

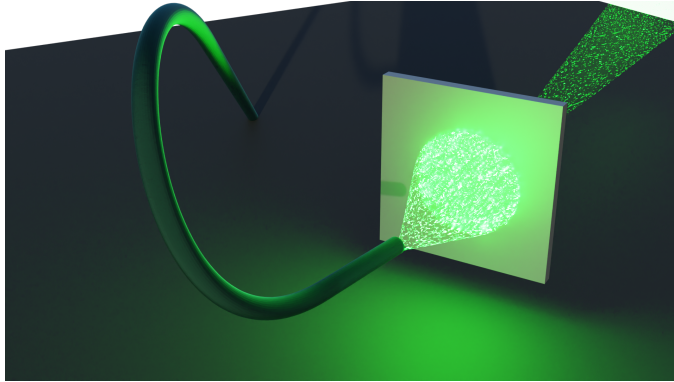


Figure 2.1: Illustration of an MMF. Coherent light coupled into the MMF produces a speckle pattern at the output (Image courtesy of Ksenia Abrashitova).

Optical fibers are dielectric waveguides designed to confine and guide light over long distances with low attenuation. Fig. 2.1 illustrates coherent light propagating through an MMF and producing a characteristic intensity pattern at the output. Practical optical fibers became feasible only in the twentieth century with advances in glass purification and fabrication techniques. Early demonstrations of image transmission through fiber bundles were reported in the 1950s, notably by Kapany et al [64]. In 1966, Charles K. Kao and George A. Hockham showed that optical losses in glass fibers were mainly caused by material impurities and argued that these losses could be reduced below 20 dB/km. This insight made practical optical communication systems feasible and laid the groundwork for modern fiber-optic technology. Kao's contribution was later recognized with the Nobel Prize in Physics in 2009 [65, 66]. Subsequent advances in fiber fabrication, particularly the development of low-loss silica fibers in the early 1970s, enabled the rapid expansion of fiber-based communication networks.

The most widely used material for optical fibers is high-purity silica glass (SiO_2), due to its excellent optical transparency in the visible and near-infrared spectral regions, its mechanical robustness, and chemical stability. The refractive index profile of the fiber is controlled by doping the silica core with materials such as germanium or phosphorus, while fluorine doping is often used to reduce the refractive index of the cladding. Optical fibers are fabricated using a preform-based drawing process, in which a macroscopic glass preform with a precisely defined refractive index profile is heated and drawn into a thin fiber. The geometric and optical properties of the resulting fiber, including core diameter, numerical aperture (NA), and modal structure, are determined by the design of the preform and the drawing conditions [67].

2.1 STEP-INDEX FIBER PARAMETERS

A conventional step-index optical fiber therefore consists of a central core with refractive index n_{core} surrounded by a cladding with a slightly lower refractive index n_{clad} , such that light is guided by *total internal reflection* at the core–cladding interface. Repeated many times along the fiber length, this confinement allows light to propagate over long distances

[68].

The ability of a fiber to capture light is quantified by its NA. From Snell's law, the NA is given by

$$NA = n_0 \sin \theta_{\max}, \quad (2.1)$$

where n_0 is the refractive index of the external medium (typically air, $n_0 \approx 1$) and $\theta_{\max}$ is the maximum acceptance half-angle. Thus, the NA and acceptance angle provide the most intuitive picture: they describe the physical cone of rays that will be guided, as shown in Fig. 2.2(a). The NA also sets the resolution limit for MMF-based imaging systems, as the speckle grain size scales with $\lambda/(2 NA)$.

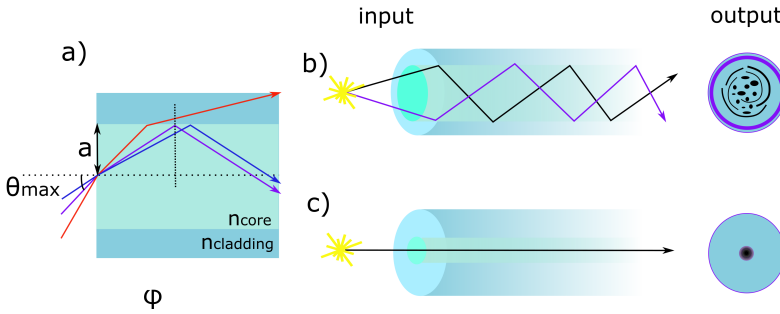


Figure 2.2: a) Geometry of a step-index optical fiber. The core of refractive index n_{core} and radius a is surrounded by cladding of refractive index n_{clad} . The NA is determined by the refractive index contrast and sets the maximum acceptance angle $\theta_{\max}$ for guided light in an external medium of index n_0 . b) An MMF supports many modes with distinct spatial profiles, while c) in a SMF, only the fundamental HE_{11} mode is supported.

The number of guided spatial modes determines the behavior of an MMF. This is conveniently described by the normalized frequency, or *V-number*,

$$V = \frac{2\pi a}{\lambda} NA, \quad (2.2)$$

which depends on the core radius a , the numerical aperture, and the operating wavelength λ . For large V , the approximate number of guided modes is

$$M \approx \frac{V^2}{2}. \quad (2.3)$$

The physics behind this formula is simple: the fiber cross-section acts like a two-dimensional box where transverse field distributions must satisfy boundary conditions. As the core radius or NA increases, more field configurations “fit,” just as more resonant states exist in a larger cavity.

A single-mode fiber (SMF) operates when the normalized frequency satisfies $V \lesssim 2.405$, so that only the fundamental HE_{11} mode propagates [68]. This mode is stable, has an approximately Gaussian intensity profile, and is relatively insensitive to bending. In contrast, when $V \gg 2.405$, an MMF supports hundreds or even thousands of guided modes (Fig. 2.2(b)). From a practical perspective, this large number of modes creates both challenges and opportunities. Modal interference leads to strong spatial scrambling, preventing direct

image transmission through the fiber. At the same time, the high-dimensional modal space can be exploited for information encoding, structured or random illumination [33, 40, 51, 69]. As a result, MMFs can be used not only as waveguides, but also as controllable complex optical systems.

2

2.2 SIMULATION OF LIGHT PROPAGATION THROUGH A MULTIMODE FIBER

Light propagation in an MMF is governed by Maxwell's equations subject to boundary conditions at the core-cladding interface, which produce a discrete set of guided eigenmodes. Each guided mode is described by a transverse field profile $\psi_m(x, y)$ and a propagation constant β_m . After coupling into the fiber, the optical field propagates as a coherent superposition of these modes, with excitation coefficients determined by the spatial profile of the input field at the fiber facet. During propagation, the modes accumulate phase at different rates, and their interference at the fiber output produces a granular intensity distribution known as speckle [15, 68]. While speckle appears random, it is fully reproducible for a fixed fiber configuration and input field, enabling controlled generation of diverse illumination patterns for imaging and sensing. In practice, however, quantitative prediction and experimental design require numerical modeling. For this purpose, we use a simulation tool that combines a semi-analytical mode solver with a propagation routine, based on the classical waveguide theory [68, 70].

Any input field at the fiber facet can be expanded into the complete set of guided modes as

$$E_{\text{in}}(x, y) = \sum_{m=1}^M a_m \psi_m(x, y), \quad (2.4)$$

Here, (x, y) denotes the transverse spatial coordinates in the plane perpendicular to the fiber axis; $\psi_m(x, y)$ represents the normalized transverse field profiles of the guided modes, indexed by m , while M denotes the total number of modes supported by the fiber at the operating wavelength. The coefficients a_m describe the excitation strength and relative phase of each mode and are determined by the overlap between the input field $E_{\text{in}}(x, y)$ and the corresponding modal field:

$$a_m = \iint E_{\text{in}}(x, y) \psi_m^*(x, y) dx dy. \quad (2.5)$$

Each mode propagates along the fiber with a propagation constant $\beta_m = n_{\text{eff},m} k_0$, where $n_{\text{eff},m}$ is the effective refractive index of the mode, $k_0 = 2\pi/\lambda$ is the vacuum wavenumber, and λ is the free-space wavelength. After propagation over a fiber length L , the output field $E_{\text{out}}(x, y)$ results from the coherent superposition of all excited modes.

$$E_{\text{out}}(x, y) = \sum_{m=1}^M a_m \psi_m(x, y) e^{-i\beta_m L}. \quad (2.6)$$

This expression shows that the observed field results from the interference of all excited guided modes, each contributing its spatial profile and accumulated propagation phase.

Equations (2.4)–(2.6) describe the general theoretical framework for modal decomposition in an MMF. To compute ψ_m and β_m , the solver evaluates the dispersion relations derived from Maxwell's equations in cylindrical coordinates with boundary conditions at the core-cladding interface [68]. Defining

$$u = a\sqrt{k_0^2 n_{\text{core}}^2 - \beta^2}, \quad w = a\sqrt{\beta^2 - k_0^2 n_{\text{clad}}^2}, \quad (2.7)$$

Here, a denotes the fiber core radius, while n_{core} and n_{clad} are the refractive indices of the core and cladding, respectively. The dimensionless parameters u and w describe the transverse field behavior in the core and the evanescent decay in the cladding.

For the scalar, weakly guiding case, the characteristic equation is

$$\frac{u J_{m+1}(u)}{J_m(u)} - \frac{w K_{m+1}(w)}{K_m(w)} = 0, \quad (2.8)$$

The functions J_m and K_m are Bessel functions of the first kind and modified Bessel functions of the second kind, respectively, which arise from the solution of Maxwell's equations in cylindrical coordinates.

2.3 SPECKLE PATTERN

When many modes are excited with comparable amplitudes, the output field of an MMF exhibits fully developed speckle. The corresponding intensity follows negative exponential statistics [15],

$$P(I) = \frac{1}{\langle I \rangle} \exp\left(-\frac{I}{\langle I \rangle}\right), \quad (2.9)$$

Here $P(I)$ denotes the probability density of measuring an intensity value I at a given spatial position in the speckle pattern, and $\langle I \rangle$ is the mean intensity averaged over many speckle grains. For fully developed speckle, the intensity fluctuations are quantified by the speckle contrast,

$$C = \frac{\sqrt{\langle I^2 \rangle - \langle I \rangle^2}}{\langle I \rangle}, \quad (2.10)$$

A speckle contrast of $C = 1$ corresponds to fully developed speckle and occurs when the illumination is fully coherent, many modes or scattering paths contribute, and no averaging is present. In this case, the intensity fluctuations are maximal and the speckle pattern exhibits strong bright and dark variations. In contrast, a speckle contrast of $C = 0$ corresponds to uniform illumination with no intensity fluctuations. This situation arises when the illumination is completely incoherent or when a large number of independent speckle patterns are perfectly averaged. In practice, this limit is not reached exactly [71].

2.4 LIGHT CONTROL THROUGH A MULTIMODE FIBER

MMFs were long known to scramble light due to mode coupling and were treated primarily as a performance-limiting factor. With the development of WFS and TM approaches, MMFs appeared as deterministic linear systems through which light can be controlled. The first demonstrations of deterministic control in MMFs appeared in 2011, when Di Leonardo and

Bianchi showed that WFS with a phase-only SLM enables diffraction-limited focusing and holographic pattern generation at the fiber output [72]. In the same year, Čižmár and Dholakia introduced full input–output calibration of an MMF by measuring its TM, demonstrating that the apparently random fiber transformation is deterministic and invertible [73]. An alternative approach based on digital optical phase conjugation was proposed by Papadopoulos *et al.*, who generated bright foci at the distal end of an MMF [74]. Conkey *et al.* demonstrated that DMD can replace slower SLMs for high-speed MMF control, enabling dynamic focusing and imaging [31]. Later works focused on understanding the limits of focusing through MMFs. Descieux *et al.* studied aberrations in MMF focusing and showed that insufficient mode control leads to distorted focal spots, deriving a practical criterion for the number of controlled segments based on the fiber V -number [75]. These advances enabled applications in biomedical imaging. Ohayon *et al.* demonstrated *in vivo* two-photon fluorescence imaging through an MMF using digital phase conjugation [76]. In the same year, Turtaev *et al.* developed a holographic MMF endoscope with micrometer-scale resolution and video-rate imaging using a DMD-based wavefront modulator [77]. Pikálek *et al.* studied wavelength-dependent effects in MMF endoscopes and introduced compensation methods to reduce chromatic and modal dispersion [78]. Gomes *et al.* achieved near-ideal focusing through an MMF by measuring both the amplitude and phase of the transmitted field, reporting that more than 96% of the delivered optical power could be concentrated into a diffraction-limited focus [79]. Despite these results, MMFs remain sensitive to environmental changes such as temperature fluctuations, mechanical stress, and fiber bending, which can strongly affect the output field. This sensitivity has motivated research on improving stability through recalibration, feedback control, and fiber designs that reduce sensitivity to external perturbations [40, 80–82]. As a result, MMFs are now widely studied for imaging and light delivery applications [56, 83–92].

Beyond biophotonics, MMFs are increasingly used in applications where control of coherence and speckle statistics is essential. We can distinguish two broad classes of methods for coherence control with MMFs: *random* methods, which suppress speckle by averaging many uncorrelated realizations without explicit wavefront control, and *precise* methods, which employ measured system response functions (e.g., transmission matrices) to engineer the output field deterministically. In random methods, speckle suppression relies on averaging many statistically independent realizations. Under standard assumptions, the speckle contrast scales as $C \propto 1/\sqrt{N}$, where N is the number of uncorrelated realizations [93]. Increasing N can be achieved by introducing spatial, temporal, angular, or spectral diversity [94]. A widely used implementation is mechanical scrambling, where moving or vibrating optical elements dynamically perturb the wavefront. Rotating diffusers and vibrating membranes generate rapidly fluctuating speckle patterns whose temporal average appears uniform [95–99]. While effective, these techniques rely on moving parts, increasing system complexity and potentially limiting mechanical stability and miniaturization. Other random approaches superpose multiple mutually uncorrelated fields using microlens arrays [100] multiple emitters, or combinations of fibers and diffusers [101], often at the cost of added components, synchronization, or multiple lasers. These methods are active. A particularly attractive random strategy for MMF-based illumination exploits wavelength sensitivity. Because different guided modes accumulate distinct wavelength-dependent phases, even sub-nanometer wavelength shifts can strongly decorrelate the MMF speckle

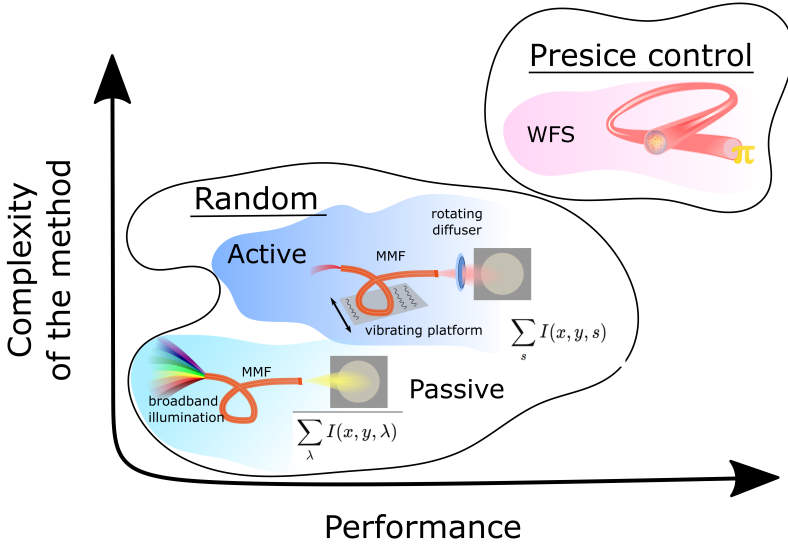


Figure 2.3: Coherence-control strategies using an MMF, illustrating the qualitative trade-off between performance and method complexity. Random approaches rely on incoherent averaging of multiple statistically independent speckle realizations: passive spectral diversity via wavelength scanning or broadband illumination, $\sum_\lambda I(x, y, \lambda)$, and active spatial decorrelation via fiber motion or input beam scanning, $\sum_s I(x, y, s)$, with an incoherent summation on the camera. Precise control is achieved, enabling deterministic WFS.

pattern [98, 102–104]. This motivates *spectral averaging*: recording speckle realizations across a wavelength sweep and summing them on the camera, $\sum_\lambda I(x, y; \lambda)$. Importantly, this approach does not have to be “active” in the mechanical sense: wavelength scanning can be implemented without moving parts (e.g., using a tunable laser sweep), providing a compact and mechanically robust route to passive speckle suppression. Pikálek et al. [78] showed that modal dispersion sets the characteristic wavelength scale over which MMF speckle remains correlated, thereby determining how rapidly independent realizations accumulate with spectral bandwidth.

In contrast to random averaging, *precise* control methods aim to deterministically engineer the coherence function of the output field. In MMFs and scattering systems, TM approaches are used to tailor second-order field statistics and, in principle, to program target coherence properties by controlling the ensemble of generated fields [105–108].

3

3

TRANSMISSION MATRIX OF A MULTIMODE FIBER: IN-LINE VS OFF-AXIS HOLOGRAPHY

Multimode fibers (MMFs) hold great promise for a variety of imaging applications. However, controlling the complex propagation of light through thousands of modes in MMFs remains a challenging task. To enable precise wavefront control, the MMF transmission matrix (TM) must be accurately measured. In this paper, we compare in-line and off-axis holography as phase retrieval techniques for TM measurement within a single optical setup. The quality of wavefront shaping is analyzed for different target shapes, including single-mode and complex free-form projections. Our findings reveal a trade-off in wavefront shaping: while in-line holography achieves superior focus quality for single-mode targets, off-axis holography excels in projecting complex-shaped targets. These experimental findings provide a basis for choosing and optimizing holographic techniques in MMF-based imaging setups.

3.1 INTRODUCTION

IN recent years, there has been increasing interest in multimode fibers (MMFs) as a promising tool for optical communication, optical trapping, optical metrology, and endoscopic imaging [69, 91]. The main challenge in MMF imaging is the complex propagation of light through thousands of interfering modes, which makes reliable information transmission difficult [109]. To address this, researchers have developed techniques such as conventional wavefront shaping [110] and digital phase conjugation [38, 52, 74, 111], enabling the manipulation of the light's spatial, temporal, and polarization degrees of freedom to optimize the fiber's output [112–114]. Furthermore, accurately characterizing the scattering properties of complex media is crucial for imaging through turbid samples [115].

A complete solution that enables full control over light propagation through complex media is to measure a full transmission matrix (TM) of the MMF [39, 41, 73, 116]. TM is a complex-valued quantity that contains both amplitude and phase information. Therefore, the choice of phase retrieval method directly affects the accuracy of the TM measurement and the effectiveness of subsequent light manipulation. Interferometric approaches such as in-line [35, 37, 117–119] and off-axis holography [36, 45, 79, 120–122] are commonly used to retrieve phase information. These methods generally have their pros and cons: in-line holography offers a simple and stable setup but can suffer from blind spots [85] due to the speckled reference, while off-axis holography avoids these artifacts at the cost of increased system complexity and sensitivity to phase drift. Active stabilization techniques with feedback control systems or computational methods may be necessary to compensate for phase drift [48, 120, 121].

Several studies have compared these approaches. Gross et al. [49] imaged a USAF resolution target using both methods and found that off-axis enables fast, single-shot acquisition but suffers from aliasing, while in-line removes unwanted terms and produces cleaner target projections. Turtaev et al. [81] compared both methods for TM-based light control using liquid crystal spatial light modulators (LC-SLMs) and digital micromirror devices (DMDs), to generate focal spots. They reported that DMDs, used with a common-path reference, achieved higher measurement speed and better focusing fidelity than LC-SLMs, which were tested with an off-axis reference. The common-path geometry also improved stability. Cižmár et al. [73] demonstrated both focal spot and complex pattern projection at the MMF output using a measured TM with the Gerchberg-Saxton algorithm, relying on iterative computational phase retrieval.

In this work, we experimentally compare two interferometric phase retrieval approaches: in-line and off-axis holography for TM measurements of an MMF, within the same DMD-based setup. In contrast to previous studies, we investigate their performance for projecting both single focal spots and complex-shaped illumination patterns. We found that off-axis holography delivers higher fidelity for complex-shape projections. In contrast, in-line holography is more robust and stable for single-spot focusing due to its common-path geometry, which reduces sensitivity to phase drift. These findings provide practical guidance for selecting an interferometric phase retrieval method in MMF-based imaging systems depending on the application requirements.

3.2 METHODS

3.2.1 TRANSMISSION MATRIX

The TM is a mathematical representation of how light propagates through complex media [41]. Although in our experiments light propagates through guided fiber modes, we measure and use the TM in the spatial basis, where the input modes n correspond to discrete spatial segments on the DMD, and the output modes m correspond to pixels on the output camera. This basis aligns with the experimental degrees of control and observation, allowing us to express the light propagation as:

$$E_m^{out} = \sum_n k_{mn} E_n^{in}, \quad (3.1)$$

where E_n^{in} denotes the complex field modulated by the n -th DMD segment, E_m^{out} is the complex field measured at the m -th camera pixel, and k_{mn} are the complex-valued elements of the TM.

3.2.2 EXPERIMENTAL SETUP

A detailed picture of the experimental setup is presented in Fig. 3.1. The input facet of the MMF is scanned with a focused laser beam with a diffraction-limited spot size, where $d_{lim} = 0.8 \mu\text{m}$, to characterize the fiber. The beam is from a single-frequency laser source (Cobolt Bolero) with a wavelength of $\lambda = 640 \text{ nm}$ and a maximum power of 300 mW. The light source has a specified linewidth of approximately 500 kHz, corresponding to a coherence length on the order of 600 meters. Only vertically polarized light filtered with a polarization beam splitter (PBS1, Thorlabs PBS251) was used to perform further measurements. The laser beam was expanded by a telescope (L1, Thorlabs, AC254-040-A-M; L2, Thorlabs, AC254-125-A-M) to increase the area that hits the DMD with a magnification of $3.12\times$ to 4 mm diameter. The DMD (Texas Instruments), driven by the DLP9001VIS module (VIALUX), with 2560×1600 micromirrors and a pitch size of $7.6 \mu\text{m}$, was used. The DMD works like a 2D diffraction grating, so the light that is diffracted has many diffraction orders. One of these orders is isolated using the pinhole (PH, Thorlabs, SM1D12C) placed in the Fourier plane of L3. The 4f-system with two lenses (L3, L4, Thorlabs, AC508-150-A-ML-Ø2) was set to image the DMD on the back focal plane of objective OBJ1 (Olympus 20×, NA = 0.25). OBJ1 was used to couple the light into the MMF. By using camera CAM1 (IDS, U3 – 3880CP-M-GL, pixel size of $2.4 \times 2.4 \mu\text{m}$, 8 bit), the input facet of the MMF is imaged for alignment purposes.

For all measurements, a step-index MMF (Thorlabs, FG050UGA) with a core radius, $a = 25 \mu\text{m}$, NA = 0.22, and a length of 30 cm was used. The number of guided modes, M , can be estimated as $M = V^2/2$, where V is the normalized frequency, $V = 2\pi a \text{NA}/\lambda$. The given MMF supports $M \approx 1450$ modes in total, or about 725 modes per polarization. After propagation through the fiber, the light is collected by objective OBJ2 (Leica 40×, NA = 0.65). The MMF output facet is imaged in camera CAM2 (IDS, U3-3070CP-M-GL, pixel size $3.45 \mu\text{m}$). Inside the MMF, polarization mixing occurs, and PBS2 (Thorlabs PBS251) is used to separate the vertical polarization, ensuring control and measurement of the same polarization state. Subsection 3.2.3 provides a detailed explanation of the implementation of off-axis and in-line holography methods.

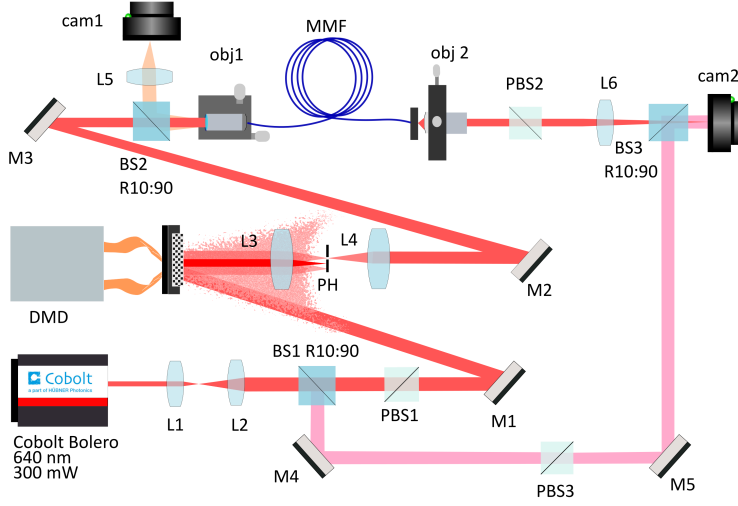


Figure 3.1: **The schematic diagram of the experimental setup.** The laser beam splits with BS1 directing 10% of the light towards mirror M4 to create a reference arm. The transmitted beam hits the DMD to generate multiple illuminations. Subsequently, the beam is coupled into the input facet of the MMF by OBJ1. The beam from the output facet is collected by OBJ2 and imaged on CAM2 using a tube lens L6. CAM1 is used to display the input facet of the MMF.

3.2.3 OFF-AXIS AND IN-LINE HOLOGRAPHY

The basic principle behind holographic reconstruction is recording interference patterns created between a reference beam and an object beam. These two beams, which are coherent with each other, combine to create an intensity distribution that contains information about the phase and amplitude of the object beam. Off-axis holography is a phase imaging technique in which the reference and object beams overlap at a small angle [123]. An interference intensity profile, $I(x, y)$, can be expressed as:

$$I(x, y) = |O(x, y)|^2 + |R(x, y)|^2 + O^*(x, y)R(x, y) + O(x, y)R^*(x, y), \quad (3.2)$$

where $R(x, y)$ and $O(x, y)$ denote reference and object fields, respectively.

The complex field $C(x, y) = O(x, y)R^*(x, y)$ can be recovered from the experimentally measured $I(x, y)$. In our setup, the beam splitter, BS1 R10:90 (Thorlabs, BS025), deflects a portion of the beam for the off-axis holography reference arm. The mirrors M4 and M5 direct the reference beam towards camera 2. To characterize the TM across hundreds of MMF modes, a series of input illuminations was projected onto the DMD. We defined the working area of the DMD as 480×480 pixels, divided into 900 segments with the size of 16×16 micro-mirrors. A binary device, such as a DMD, creates a sinusoidal grating by applying the Lee holography method [32]. First, the desired wavefront is converted into a binary hologram using the Lee method, where the phase and amplitude of the wave are represented by the position and width of the fringes. Each binary hologram is then formatted according to the pixel grid (segment) of the DMD. For the experiments, a DMD segment contains a sinusoidal grating with an encoded phase pattern ($\varphi = 0$). All other segments are

turned off. This means that only one segment reflects the beam towards the MMF, while the remaining segments deflect the wavefront.

By sequentially activating the segments, we scan the input facet of the fiber as illustrated in Fig. 3.2(a) and obtain a set of interference patterns on camera 2. A total of 900 interference patterns were recorded, which formed a stack of measurements over the spatial domain. The complete stack can be expressed as: $\mathcal{I}(x, y, n) = \{I_1(x, y), I_2(x, y), \dots, I_{900}(x, y)\}$, where $\mathcal{I}(x, y, n)$ is a three-dimensional dataset with dimensions corresponding to the spatial coordinates (x, y) and the image index n . By applying the Fourier transform to $\mathcal{I}(x, y, n)$, the cross-terms can be isolated as distinct sidebands in the frequency domain, as shown in Fig. 3.2(a). After selecting one sideband using a spatial filter mask, the inverse Fourier transform is applied to retrieve the complex field, from which the amplitude and phase information are extracted. Now, the complex-valued image matrices can be reshaped into the TM as shown in Fig. 3.2(c).

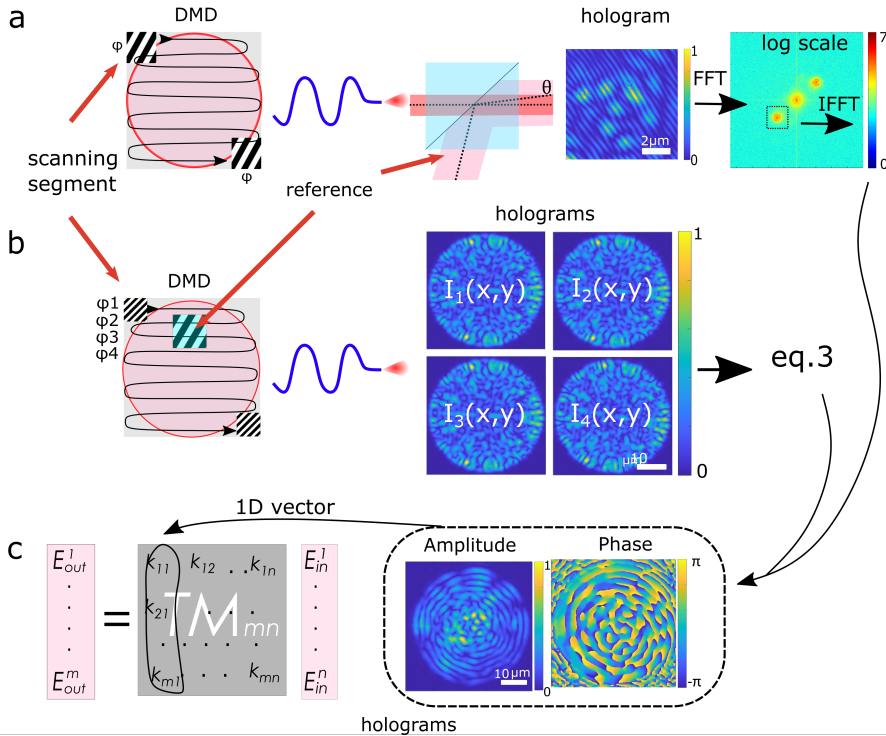


Figure 3.2: **Schematic of complex field retrieval from holographic measurements.** (a) Off-axis holography. Left: example DMD pattern where a single segment is active, set constant phase, and is changing the position across the DMD active area. Middle: resulting interference pattern of the MMF output and the reference plane wave. Right: sideband isolation in the Fourier domain and inverse transform to retrieve the complex field. (b) In-line holography. Left: DMD pattern showing a static internal reference region placed within the DMD working area and a modulated signal segment. The stack of resulting intensity patterns $\mathcal{I}(x, y, n)$ is processed to extract the complex output field. (c) From right to left: complex-valued output images corresponding to different input modes are flattened and form a TM.

In-line holography uses a common beam path for both the reference and signal beams, reducing the complexity of the optical setup and alignment requirements. It generally provides a stability advantage over off-axis holography, as the reference and signal beams share the same optical path. This minimizes relative phase drift caused by vibrations or environmental disturbances. In-line holography is a technique that involves creating a series of interference patterns using a predefined object beam with phase shifts.

In our implementation, the reference beam is generated internally by activating a small 20×20 micromirror region within the 480×480 active area of the DMD. This patch provides a fixed, known phase reference set to “0”, as shown in Fig. 3.2(b). The remaining DMD segments sequentially scan the object field across the full working area; during this process, the scanning segment (16×16 micromirrors, $122 \times 122 \mu\text{m}$) partially overlaps the reference region but never fully covers it, ensuring that the reference is always present in the recorded interference pattern. Because both reference and object fields co-propagate through the MMF, they experience identical modal dispersion and mode mixing. At the fiber output, their interference produces a speckle pattern, from which the complex optical field is reconstructed using the four-step phase-shifting method [124]. However, since the reference itself is scrambled by the MMF, it emerges as a random speckle field rather than a smooth wavefront, with a highly non-uniform and spatially varying phase.

Unlike off-axis holography, where each DMD segment has a fixed phase, in the in-line case, the relative phase shift between the reference and control sections is varied in four steps: $\phi_1 = 0$, $\phi_2 = \pi/2$, $\phi_3 = \pi$, and $\phi_4 = 3\pi/2$. For each of the 900 input positions, four interference patterns were recorded, resulting in a four-dimensional dataset: $\mathcal{I}(x, y, n, p) = \{I_{n,p}(x, y)\}$, where (x, y) are the spatial coordinates, image index n , and phase step index p . The “four-phase method” was used to retrieve the amplitude and phase. The complex field $C(x, y)$ can be reconstructed as:

$$C(x, y) = \frac{I_1(x, y) - I_3(x, y)}{4} + i \frac{I_4(x, y) - I_2(x, y)}{4} \quad (3.3)$$

3.2.4 NUMERICAL SIMULATIONS

To simulate light propagation through the MMF, a semi-analytical scalar mode solver was implemented under the weak-guidance approximation [125]. The simulation consists of two steps: calculation of all guided linear-polarized (LP) modes and their propagation constants, and modal decomposition and propagation of an arbitrary input field. For an input field $E_{\text{in}}(x, y)$, the output field after a fiber length L is given by:

$$E_{\text{out}}(x, y, L) = \sum_{n=1}^{N_{\text{modes}}} \alpha_n E_n(x, y) e^{-i\beta_n L}, \quad (3.4)$$

where $E_n(x, y)$ is the transverse profile of the n -th guided LP mode, β_n is its propagation constant, and N_{modes} is the total number of guided modes. The coupling coefficients

$$\alpha_n = \iint E_{\text{in}}(x, y) E_n^*(x, y) dx dy \quad (3.5)$$

represent the overlap between the input field and each mode [68].

The guided-mode set $\{E_n, \beta_n\}$ is obtained by solving the characteristic equation for a circular step-index fiber core:

$$\frac{u J_{m+1}(u)}{J_m(u)} = \frac{w K_{m+1}(w)}{K_m(w)}, \quad (3.6)$$

with

$$u = a \sqrt{k_0^2 n_{\text{core}}^2 - \beta^2}, \quad w = a \sqrt{\beta^2 - k_0^2 n_{\text{clad}}^2}. \quad (3.7)$$

Here a is the core radius, n_{core} and n_{clad} are the core and cladding refractive indices, $k_0 = 2\pi/\lambda$ is the vacuum wavenumber, J_m and K_m are Bessel and modified Bessel functions of order m , and m is the azimuthal index. The quantities u and w are dimensionless transverse wave-number parameters. Once the modal basis is obtained, arbitrary input fields are propagated using Eq. 3.4.

Both in-line and off-axis holography are simulated using the same mode solver and identical fiber parameters and spatial basis. A single input basis field is a 16×16 pxls ($1.7 \times 1.7 \mu\text{m}$) segment at the fiber input facet. The output field is sampled on a 480×480 pixel frame. TM_{off} is obtained by propagating each input segment with “0” phase through the MMF and extracting the complex output fields directly, corresponding to an ideal, noise-free off-axis measurement. For the in-line simulation, each input basis segment is sequentially encoded with four phase shifts ($0, \pi/2, \pi, 3\pi/2$), and each pattern is propagated through the MMF. The resulting noise-free intensity distributions at the MMF output are calculated. The complex field is reconstructed using Eq. 3.3, and the resulting complex fields for all basis vectors constitute TM_{in} .

Once TM_{off} and TM_{in} are obtained, all subsequent steps are identical for both methods. The desired output pattern (target, 480×480 pixels) is vectorized, multiplied by the conjugate transpose of the respective TM to compute the required phase-only input field, which is numerically propagated through the MMF to obtain the final output intensity distribution.

3.3 RESULTS

3.3.1 STABILITY CHARACTERIZATION

Wavefront engineering on the MMF output requires high interferometric stability. The in-line configuration provides inherent stability due to its common-path geometry, whereas the off-axis approach relies on an external reference and is therefore more sensitive to vibration and air turbulence. To quantify these effects, stability measurements were performed for both configurations using the same optical setup and a fixed acquisition rate of 120 Hz for off-axis and 30 Hz for in-line holography, respectively. To allow a direct comparison between the two phase-retrieval methods, the stability traces are evaluated over the same time window of 7.5 s.

For in-line stability measurements, a sequence of four phases was projected in a single input position of the DMD segment, which is described in the Methods section. A separate DMD segment has been used as a zero-phase reference. All other DMD segments were switched off. A total of 3600 intensity patterns on the MMF output was recorded. The reconstruction procedure described above was applied to extract 900 light fields (both phase and amplitude). The area in pixels selected for analysis corresponds to the diffraction limit of the MMF, which is $d_{\text{lim}} = \lambda/(2 \text{NA}) \approx 1.45 \mu\text{m}$. The phase mean value with the $1.45 \mu\text{m}$

$\times 1.45 \mu\text{m}$ region was calculated as a function of time. Temporal phase fluctuations in a diffraction-limited region at the MMF output, measured using in-line holography, are shown in blue in Fig. 3.3(a). The standard deviation of 900 phase measurements in 30 seconds is calculated as 0.03 radians.

A similar measurement procedure is applied for off-axis holography. A constant zero phase was projected onto the same input position of the DMD segment. All other segments were turned off. A total of 900 interference patterns between the field at the fiber output and the external reference have been recorded. The phase profiles have then been calculated as discussed in the Methods section. The standard deviation of 900 phase measurements over 7 seconds is equal to 0.04 radians. The results are presented in Fig. 3.3(a) (red curve). Although in-line holography measurements have the potential for greater stability, our results show very similar performance for both the in-line and off-axis approaches.

3.3.2 PROJECTING SINGLE-MODE TARGETS

To control the intensity distribution at the MMF output, the first experiment focused on generating a “single-mode” projection with a size of the diffraction-limited spot. A target field E_{out} was defined as a $0.345 \times 0.345 \mu\text{m}$ region (a 4×4 pixel area) at the desired camera-plane position. The corresponding input field was obtained by backpropagating the target through the conjugate transpose of the measured TM:

$$E_{\text{in}} = \mathbf{T}^\dagger E_{\text{out}}. \quad (3.8)$$

The chosen target area is much smaller than the diffraction limit of our MMF ($1.45 \mu\text{m}$), ensuring that the desired spot occupies a single spatial mode.

To quantify focusing performance, a power ratio (PR) metric was introduced, defined as the ratio of the intensity within the focal spot at its half-maximum level, I_{foc} , to the total transmitted intensity, I_{total} :

$$\text{PR} = \frac{I_{\text{foc}}}{I_{\text{total}}} \quad (3.9)$$

where I_{foc} is calculated by integrating the signal within a circular area with a radius equal to the full width at half maximum (FWHM) of the theoretical diffraction limit of the MMF.

First, we measure the full TM of the MMF using the off-axis holography approach. The TM_{off} consists of 419904×900 elements, where 419904 represents the size of the flattened camera image of 648×648 pixels, and 900 corresponds to the number of DMD segments. A set of 20 phase masks is used to sample different locations within the fiber core, each mask producing a single diffraction-limited focal spot after propagation through the MMF. Each phase mask is projected onto the DMD, and the resulting intensity at the MMF output facet is recorded. The incoherent sum of 20 individual focal spots is presented in Fig. 3.3(b). The average PR of focal spots calculated using off-axis TM measurements is $\text{PR}_{\text{off}} = 41.4 \pm 2.6\%$. The theoretical limit of PR corresponds to a perfect diffraction-limited focus, assuming ideal wavefront control and no background noise or mode-mixing artifacts. Although our approach did not achieve the theoretical maximum PR, previous work [79] has demonstrated that $\text{PR} = 96\%$ is achievable, providing a reference to optimize imaging performance under various experimental conditions.

The same experimental setup, with the reference arm blocked, is used to measure the TM_{in} (419904×900) of the MMF via in-line holography. A set of 20 phase masks is then

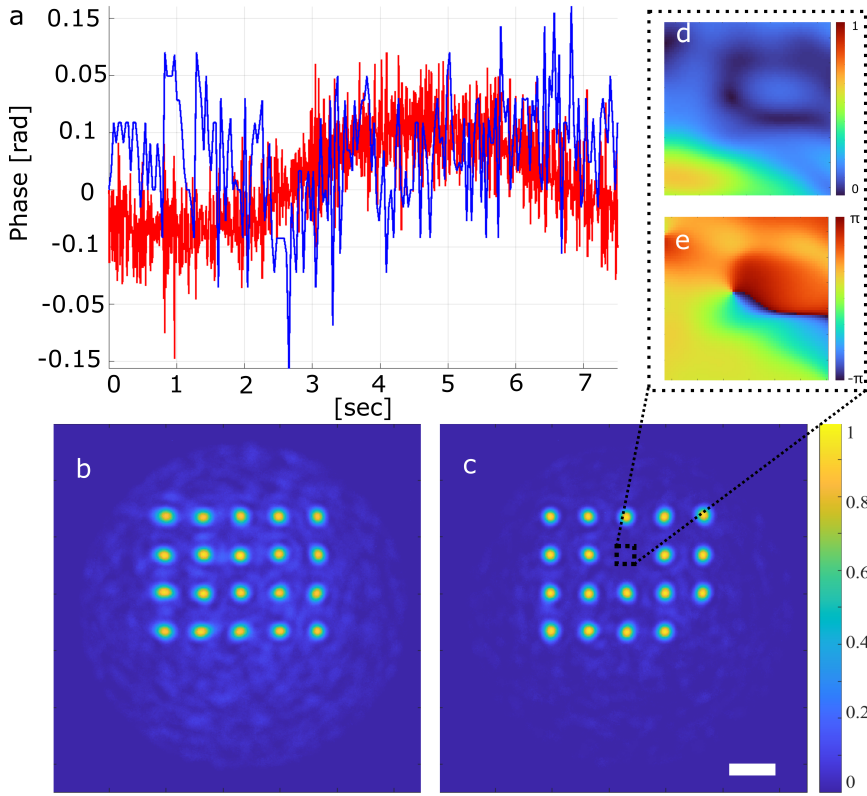


Figure 3.3: **Experimental results.** (a) Temporal phase stability of a single speckle grain at the MMF output measured over a 7 s interval for the in-line (blue) and off-axis (red) configurations. (b,c) Incoherent sum of 20 images of focal spots at the MMF output facet optimized with off-axis (b) and in-line (c) approaches. The scale bar is 10 μm . (d,e) Reconstructed intensity (d) and phase (e) distributions of the zoomed-in region ($2 \times 2 \mu\text{m}$) corresponding to the singularity.

retrieved, each corresponding to an optimized diffraction-limited focal spot at predefined positions on the MMF output facet. Each mask is projected onto the DMD, and the resulting intensity patterns at the MMF output are recorded. The incoherent sum of 20 individual projections is presented in Fig. 3.3(c). The average PR of the focal spots optimized using in-line TM measurements, $\text{PR}_{\text{in}} = 56.4 \pm 17.8\%$.

The large standard deviation is primarily caused by phase singularities in the reference field, at points where the amplitude vanishes and the phase becomes undefined. At these locations the interference contrast collapses, preventing reliable phase retrieval. The resulting “blind spots” visible as dark speckle regions in the zoomed-in images in Fig. 3.3(d,e) degrade the fidelity of the TM, particularly for specific rows corresponding to those affected regions. This effect is a known limitation of in-line holography with speckle references, but it can be reduced by using multiple internal references or advanced calibration procedures [85, 126], which improve focus quality and suppress the influence of blind spots. Excluding these

“blind-spot regions”, in-line holography achieves a power ratio $PR_{in} = 61.8 \pm 6.5\%$, which is approximately 50% higher than the value obtained for off-axis holography under the same experimental conditions.

3.3.3 PROJECTING COMPLEX FREEFORM TARGETS

In the second set of experiments, we investigate the projection of complex, arbitrary-shaped targets through an MMF using different approaches to measuring TM. A binary π -shaped intensity distribution was chosen as the desired output pattern, and the corresponding optimized input fields were computed using the experimentally measured transmission matrices TM_{off} and TM_{in} . The resulting output intensities are shown in Fig. 3.4. Using TM_{off} , the projected pattern (Fig. 3.4a) closely reproduces the target shape and maintains clear boundaries. In contrast, projection using TM_{in} (Fig. 3.4c) fails to recover the structure of the π -shape: the output lacks continuity and contains severe distortions. This result is markedly different from the performance observed for a diffraction-limited spot projection.

The non-homogeneous intensity distribution and rapidly varying phase of the in-line reference results in a discontinuous shape. Rapid phase changes within the target area, which in the case of complex shapes can be much larger than the diffraction limit, also disrupt the image projection. This results in phase discontinuities across the reconstructed field, making it impossible to reproduce large-scale structures.

Numerical simulations introduced in Section 3.2.4 confirm these observations. The simulated projections are shown in Fig. 3.4(b,d). Wavefront shaping based on TM_{off} produces an accurate result, while TM_{in} -based approach fails. This agreement between experiment and simulation demonstrates that the performance difference arises from the TM acquisition procedure itself, not from experimental noise or instabilities.

The off-axis simulation with TM_{off} produces a clear and accurate reconstruction, while the image projected with the in-line TM_{in} has no distinct boundaries or clearly defined shape. This agreement between experiment and simulation demonstrates that the performance difference arises from the TM acquisition procedure itself, not from experimental noise or instabilities.

To mitigate the limitations of the in-line holography, we applied a reference-field normalization to TM_{in} , correcting the measured object field $C(x, y)$ using the amplitude and phase of the reference field $R(x, y)$:

$$O(x, y) = \frac{C(x, y)}{|R(x, y)|} e^{i\phi_R(x, y)}. \quad (3.10)$$

Figure 3.4(e) shows the simulated target projection after applying the reference-field correction to TM_{in} . Once the influence of the non-uniform internal reference is removed, the in-line method produces an output pattern that closely matches the result of the off-axis approach (compare Fig. 3.4(b,e)). This convergence is expected, as both methods ultimately operate under the same physical constraints: the use of phase-only modulation, the finite number of guided modes supported by the MMF, and the diffraction limit imposed by the fiber’s NA. However, the correction requires independent measurement of the complex reference field, which removes the experimental simplicity and stability associated with in-line holography.

To quantify the fidelity of the target projection, the Pearson correlation coefficient (PCC)

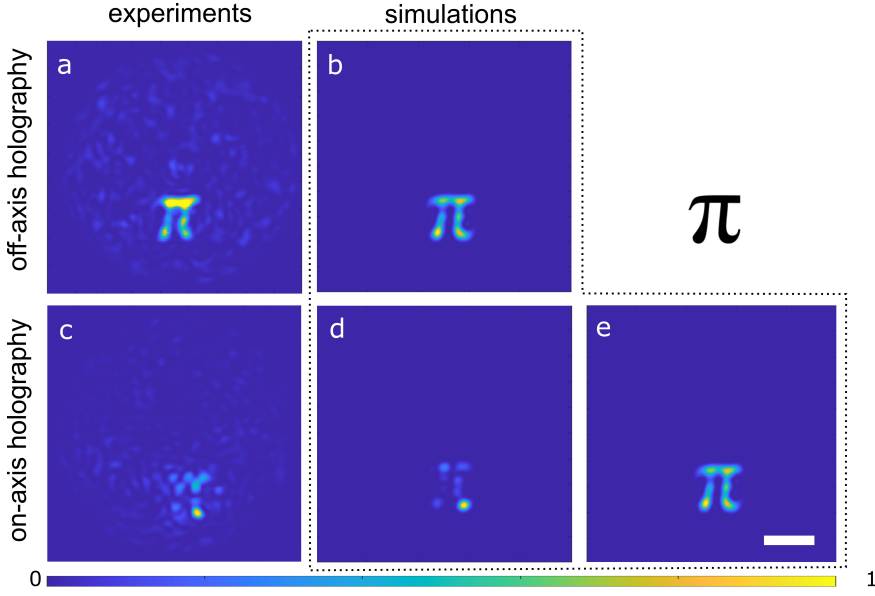


Figure 3.4: **Intensity distribution created at the output facet of the MMF:** experimental (a, c) and simulated (b,d,e) data. Results were obtained using off-axis (a,b) and in-line (c,d,e) holography. The binary target is shown in the top right. (d, e) Simulated result for in-line holography without (d) and with (e) reference field correction. The scale bar is 10 μm .

was used,

$$r = \frac{\sum_i (T_i - \bar{T})(R_i - \bar{R})}{\sqrt{\sum_i (T_i - \bar{T})^2} \sqrt{\sum_i (R_i - \bar{R})^2}} \quad (3.11)$$

which measures similarity between the target intensity T and the projected intensity R , i is the index of all pixels within the chosen ROI. Experimentally, TM_{off} achieved $r = 0.51$, whereas TM_{in} produced only $r = 0.15$. Simulations followed the same trend, yielding $r = 0.82$ for TM_{off} and $r = 0.67$ for TM_{in} . After correction, $\text{TM}_{\text{incorr}}$ improved to $r = 0.79$, confirming that the primary source of distortion in the in-line projections is the spatial structure of the internal reference.

In summary, off-axis holography enables accurate projection of complex freeform targets through an MMF, while in-line holography suffers from intrinsic reference-related distortions that prevent coherent reconstruction of extended shapes. Although reference correction can partially restore performance, TM_{off} measurements remain the more robust approach for complex pattern generation.

3.4 DISCUSSION

To further investigate the relative performance of in-line and off-axis holography for projecting complex targets through an MMF, we performed a systematic numerical study using a set of 10 grayscale MNIST digits (0-9). We simulated off-axis and in-line TM

measurements using identical MMF parameters (core diameter $50\ \mu\text{m}$, $\text{NA} = 0.22$, fiber length $1\ \text{m}$) as described in Section 3.2.4. Projection fidelity was quantified using PCC (Eq. 3.11), evaluated within a fixed region of interest (ROI).

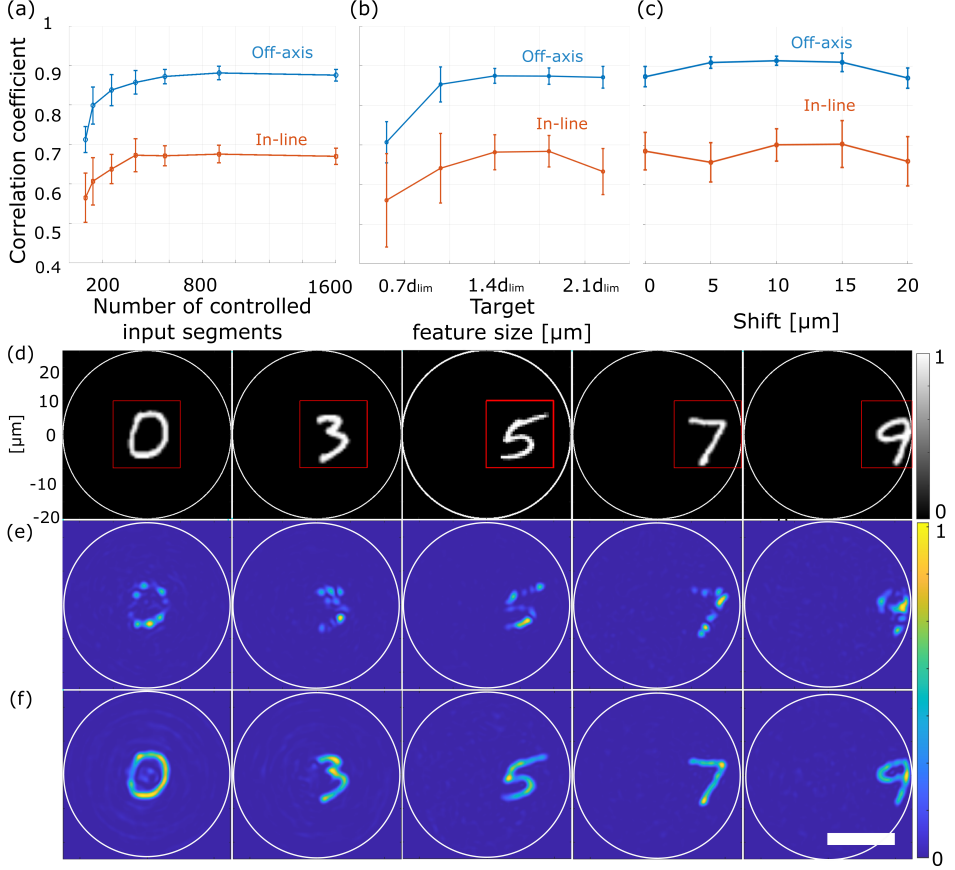


Figure 3.5: Systematic comparison of in-line and off-axis holography methods for wavefront shaping through an MMF for the MNIST dataset images as targets. (a-c) PCC between the target and projected images as a function of the number of controlled input segments (a); the target feature size (b); lateral target displacement across the fiber facet (c). d_{lim} denotes the diffraction-limited spot size of the MMF. (d) Examples of target intensity patterns. The target position is shifted from the center from 0 to $20\ \mu\text{m}$ in $5\ \mu\text{m}$ steps (from left to right). The red square indicates the $20 \times 20\ \mu\text{m}$ ROI used for PCC evaluation. (e, f) Intensity distributions on the MMF output facet obtained by wavefront shaping with TM_{in} (e) and TM_{off} (f). The white circles indicate the MMF core area. Scale bar: $20\ \mu\text{m}$.

(i) Number of controlled modes. The first set of simulations examined how the projection quality changes with the number of controlled DMD segments. Grayscale digits 1, 3, 5, 7, and 9 with a size of $20 \times 20\ \mu\text{m}$ positioned at the center of the fiber facet were used as targets. For each digit, two optimized input phase patterns were calculated using TM_{in} and TM_{off} , respectively. We repeated simulations for different total numbers of controlled

input segments from 100 to 1600.

The results are presented in Fig. 3.5(a). As the number of controlled segments increases, the correlation between the desired target shape and actual projection rises and eventually saturates once the number of controlled segments approaches the fiber's guided-mode capacity, which is ~ 1450 guided modes. Off-axis holography achieves $r = 0.87 \pm 0.02$, whereas in-line holography saturates near $r = 0.67 \pm 0.02$.

(ii) Target feature size. In the second set of simulations, we evaluated how target projection performance using TM_{in} and TM_{off} methods depends on the feature size of the target. We used 1600 controlled segments and ten grayscale MNIST digits (0–9) placed at the center of the MMF output facet as targets. We repeated simulations for different target sizes varied from $0.5 \times d_{\text{lim}}$ to $2.0 \times d_{\text{lim}}$, and ROI scaled proportionally ($8 \times 8 \mu\text{m}$ to $32 \times 32 \mu\text{m}$). The effective feature size of each digit was quantified using the FWHM of its normalized autocorrelation. $C(m, n) = \sum_i \sum_j A(i, j) A(i + m, j + n)$, where i and j enumerate pixel coordinates of the images, and (m, n) denote the spatial shifts.

The results are presented in Fig. 3.5(b). For feature sizes below the diffraction limit d_{lim} , both methods yield low correlation. Once the feature size exceeds d_{lim} , the fidelity increases sharply. Off-axis holography reaches $r = 0.87 \pm 0.02$, while in-line holography reaches only $r = 0.68 \pm 0.05$.

(iii) Target position across the fiber facet. To evaluate how projection quality varies across the fiber facet, the $20 \times 20 \mu\text{m}$ targets were shifted laterally five times in $5 \mu\text{m}$ steps from the center of the fiber core toward the cladding, as presented in Fig. 3.5(c). For each position, the optimized input phase mask was calculated. The corresponding intensity patterns at the MMF output obtained using wavefront shaping with TM_{in} and TM_{off} are presented in Fig. 3.5(d) and (e), respectively. As shown in Fig. 3.5(c), the off-axis method maintains high and stable performance with a slight reduction at the center and near the cladding. The in-line method shows lower performance and higher noise

(iv) Grayscale vs. binary targets. Finally, we tested the performance quality for binary and grayscale targets. Optimized phase masks were calculated for grayscale and binarized versions of five MNIST digits (1–5) projected at the center of the fiber output facet. The performance of off-axis holography shows $r = 0.88 \pm 0.02$ and $r = 0.84 \pm 0.04$ for grayscale and binary targets, respectively. The in-line method followed the same trend but with lower fidelity of $r = 0.66 \pm 0.06$ and $r = 0.63 \pm 0.07$. Grayscale targets consistently performed better because their smoother intensity variations contain fewer high-frequency components, making them easier to reproduce and less sensitive to noise.

Overall, off-axis holography demonstrates better performance in projecting a variety of complex targets at multiple locations at the MMF output facet, whereas the in-line method generally suffers from random phase variations. We note that simulations demonstrate better performance than experimental results, as shown in 3.4. The numerical model assumes ideal, noise-free conditions and the same linear polarization at the input and output of the MMF, whereas in the experiments, light scrambling leads to two orthogonal polarizations at the MMF output. Because we control only one, the wavefront shaping quality is reduced.

CONCLUSION

Our findings reveal a key trade-off in wavefront shaping through an MMF using TM characterization: while in-line holography provides better focus quality and stability for

single-mode diffraction-limited targets, off-axis holography excels in projecting complex-shaped patterns. In-line holography benefits from a common-path geometry, where both signal and reference fields co-propagate through the same MMF. This configuration ensures minimal phase drift and excellent stability. The issue of phase singularities in the internally generated reference field is a well-known limitation and has been addressed in prior work using multiple internal references. These techniques can mitigate “blind spots” and enhance single-spot focusing efficiency. However, even when singularities are avoided, the inherently non-uniform and rapidly varying phase of the reference field prevents globally consistent phase retrieval. Consequently, in-line holography cannot reliably reproduce extended or complex-shaped targets.

Off-axis holography, in contrast, uses a spatially flat external reference that bypasses the fiber and avoids modal scrambling. However, this approach comes with trade-offs: the optical setup is more complex, requiring separate reference and object paths, which increases sensitivity to environmental disturbances such as vibrations and air turbulence. The off-axis reference, traveling outside the MMF, picks up independent phase noise. These independent phase fluctuations introduce temporal phase noise, which is especially detrimental when projecting diffraction-limited spots, where precise phase alignment is essential for optimal focusing. As a result, off-axis holography, while ideal for projecting complex targets, is inherently less stable for single-spot generation than the in-line configuration. Understanding this trade-off is essential for optimizing MMF-based imaging, beam shaping, and signal delivery across applications in metrology, telecommunications, and biomedical optics.

4

SWEPT-SOURCE MULTIMODE FIBER IMAGING

4

High-resolution compressive imaging via a flexible multimode fiber is demonstrated using a swept-laser source and wavelength dependent speckle illumination. An in-house built swept-source allowing for independent control of bandwidth and scanning range is used to explore and demonstrate a mechanically scan-free approach for high-resolution imaging through an ultrathin and flexible fiber probe. The computational image reconstruction is shown by utilizing a narrow sweeping bandwidth of < 10 nm while acquisition time is decreased by 95% compared to conventional raster scanning endoscopy. Demonstrated narrow-band illumination in the visible spectrum is vital for the detection of fluorescence biomarkers in neuroimaging applications. The proposed approach yields device simplicity and flexibility for minimally invasive endoscopy.

4.1 INTRODUCTION

IN the development of smaller and minimal invasive endoscopes, ultra-thin fibers play a vital role. Many fiber-based endoscopy configurations have been proposed: bundles [127–129], fibers with miniaturized optics [130, 131], multi-core fibers (MCF) [132, 133] and multi-mode fibers (MMF) [84, 86, 88, 134, 135]. MCF or MMF provide imaging in hard-to-reach places by creating spatially resolved light patterns at the distal fiber end and collecting the signal from the sample. Usually, a monochromatic beam is sequentially scanned across the fiber input facet generating independent illumination patterns at the sample plane. The patterns illuminate the sample and the signal is collected and measured via a “bucket” detector. The single-pixel configuration allows for the exploration of compressive sensing algorithms, by using a sub-Nyquist set of patterns and the computational super-resolution reconstruction of the object [56, 136–142].

However, the crucial component in all those configurations is the need for a light scanning or wavefront shaping device to create desired illumination patterns at the fiber output. Commonly, either galvanometric mirrors [139], a spatial light modulator (SLM) [40, 135] or a digital micromirror device (DMD) [81, 143] are used. Choudhury et al. reported on using a single mode fiber (SMF) to sequentially illuminate the cores of a MCF, by mounting the proximal end of the MCF onto a computer controlled stage [133]. Spatial raster scanning or spatial wavefront shaping have many drawbacks including complexity of the setup, low speed, and mechanical instability. Here we explore a new way to generate dynamic illumination patterns for MMF imaging.

The wavelength of light can be used as an additional degree of freedom to control the spatial profile at the fiber output. Different wavelengths (optical frequencies) have different speed while propagating through the waveguide and therefore have different phase delays between guided modes of the MMF on the output [144, 145]. This property enables unique wavelength dependent speckle patterns at the output facet. Single-pixel imaging via compressive sensing and wavelength dependent scattering in TiO_2 layer has been recently demonstrated [146]. A similar concept using an all-fiber setup was proposed by Kubota et al. in [147]. However, no data on probe’s flexibility and imaging robustness have been reported. Moreover, in these works, near infrared (NIR) illumination was used, which makes the demonstrated systems not suitable for future applications in fluorescent imaging, since most fluorophores operate in the visible wavelength range.

Here we demonstrate an ultra-thin and flexible probe for high resolution fiber imaging in the visible domain using a custom-made swept laser source and a combined singlemode-multimode probe. Different samples have been visualised and investigated. Robustness of the imaging approach was tested by doing calibration measurements prior imaging with subsequent fiber movements mimicking real application endoscopic movement. The proposed approach does not rely on any spatial raster scanning or wavefront shaping system and can potentially provide super-resolution endoscopic imaging.

4.2 METHODS

4.2.1 EXPERIMENTAL SETUP

A swept-source, based on a super-continuum laser (Leukos Rock 400, 4W) and a custom-made monochromator was built allowing for an access to multiple degrees of freedom,

such as scanning speed, step size, spectral bandwidth of each illumination pattern and the total bandwidth. The experimental setup is presented in Figure 4.1a). The near-infrared wavelength range of the pump light was filtered out via two low-pass filters (Thorlabs, DMLP900) and a band-pass filter (Thorlabs FESH0750) such that only visible (VIS) light in the range from 400 to 750 nm was transmitted further. The beam was expanded by a factor of 8.33, using two achromatic lenses ($f = 30$ mm and 250 mm). The expanded VIS beam hits a reflective grating (Thorlabs GR50-1205 rules, 1200 l/mm with 500 nm blaze) mounted on a high-resolution rotational stage (Physics Instruments V-611.998061, connected to a C-891.130300 PIMag controller). The stage can be rotated in $1 \mu\text{rad}$ step sizes (equivalent to 0.000057°) allowing for precise wavelength selection. The first order fanned-out light reflected from the grating was focused via a concave mirror (Thorlabs, CM254-750-E02, $f = 750$ mm) onto the tip of a single-mode fiber (SMF, Thorlabs, P1-460Y-FC-1, core diameter $4 \mu\text{m}$), which acts as a pinhole. Based on the components, the theoretical diffraction limited resolution of the monochromator is 0.02 nm. The narrow wavelength bandwidth used for illumination leads to relatively low intensities of the pump light. High-power tunable laser sources could be used for future applications, as our proof-of-principle experiments demonstrate that a relatively small scanning range allows for imaging.

The SMF output was sturdily mounted on a 2D x - y stage opposite to the input facet of a sturdily mounted multimode fiber (MMF, Thorlabs M42L01, core diameter $50 \mu\text{m}$, length: 1 m). The x - y stage allowed to vary light coupling parameters. We investigated different coupling positions, from the center to the edge of the fiber and conclude that a slight off-center position give the best speckle patterns, in terms of homogeneous distribution of speckles. The results can be seen in Suppl. Figure S1. The close distance of less than $50 \mu\text{m}$ between SMF and MMF ensures that most of the light from the SMF is coupled into the MMF even for off-center positions.

To calibrate the custom made monochromator, a spectrometer (Ocean Insight, OceanHDX, max. resolution 0.7 nm) was placed at the distal end of the SMF. The output spectrum measurements at different rotation angles were taken. The data was linearly fitted, as can be seen in Figure 4.1b). Since the monochromatic output wavelength is linear related to the rotational position of the grating, a rotation of 1° equals a wavelength shift of around 27.5 nm. That means, a rotation of 0.001° (which is within the resolution limits of the rotation stage), allows a wavelength shift of 0.027 nm (which is matching the resolution of the monochromator) and similar to the resolution achieved by Redding et al. using a MM fiber for low-cost spectroscopy [148].

A large-beam achromatic fiber collimator with an adjustable focus (Thorlabs, C40FC-A) or a fiberport (Thorlabs PAF2-A4A) projected the light onto the sample with a magnification of $M = 11.1$ or $M = 44$, respectively. The magnified diffraction limit has been calculated as $M \cdot \lambda / (2\text{NA})$, where $\lambda = 500$ nm is the smallest wavelength used. A R70:T30 beam splitter (Thorlabs BS022) directed the reflected beam towards the sample and the transmitted beam towards a camera (Basler a2a1920-160um) to record the speckle patterns. An avalanche photodiode (APD, Thorlabs APD440A2) in combination with a collimating lens or a camera (Basler acA1300-200um) was used to measure the total intensity transmitted through the sample. The camera mounted close to the back side of the sample operates as a large area detector increasing measurement sensitivity. During the processing step, the camera image was integrated, and results in one intensity value per related speckle pattern, as truly for a

single pixel detector. The intensities of the speckle patterns vary with wavelength, as can be seen in Suppl. Fig. S2(d). These wavelength-dependent intensity variations have been taken into account because both the illumination matrix $\mathbf{A}$ acquired during the calibration and the intensity vector $\mathbf{b}$ recorded during the measurements are linearly dependent on the corresponding intensities.

The grating on the stage was rotated over a scanning range of one degree. At each stage position, the software waits 2 ms before a camera acquisition was triggered. The binary transparent samples, "two dots", "zero digit" and "three bars" were in-house made photolithographically objects etched onto a standard-size aluminum sputtered microscope slide. The "resolution target bars" sample was part of a negative resolution target (Thorlabs R2L2S1N). During experiment, the samples were mounted in a x - y sample holder (Thorlabs, XYFI/M), which was additionally mounted on a linear stage to obtain z -direction displacement.

4

4.2.2 IMAGING PROTOCOL

The imaging and reconstruction principle is depicted in Figure 4.1c. For each wavelength, a unique speckle pattern was passively created via the light scrambling in the MMF. The examples of speckle patterns for 15 different wavelengths between 514 to 541.5 nm in steps of 1.83 nm are shown in Suppl. Figure S2a). The cross-correlation graph and correlation histogram for all the experimentally measured speckle patterns are shown in Suppl. Figure S2b) and c), respectively. For each color coded speckle pattern, projected on a sample, the camera acquired an image and the detector behind the sample recorded the total transmitted intensity. Background measurements were not taken.

The acquired stack of speckle images and related intensities data were loaded and processed in Matlab 2022a (MathWorks). The speckle images were pre-processed by extracting their circular region of interest and by zero padding the remaining corners. For image reconstruction, the two different methods Ghost Imaging (GI) and Compressive Imaging (CI) were used. Ghost imaging, as in the single pixel imaging approach reconstructs an object, $O(x, y)$, as the weighted sum of speckle pattern (SP_i) with the coefficients calculated as subtraction of the mean intensity (b_{mean}) from the speckle related intensity (b_i) measurement [149]:

$$O(x, y) = \frac{1}{M} \sum_{i=1}^M (b_i - b_{mean}) \times SP_i(x, y), \quad (4.1)$$

where M is the number of measurements. Thus, each bucket measurement b_i is the overlap between the object and the illumination pattern. The GI can be seen as a vector projection of the object transmission function over M different random vectors SP_i . Achieving a good contrast, requires in practice $M \gg N^2$ patterns, which is not given here, therefore the GI reconstruction could be very poor or not given at all [150].

Compressive Imaging reconstruct a sample from a series of measurements by finding solutions to underdetermined linear systems using the sparsity constrain. Among several popular regularization algorithms available we chose to use TVAL3 [151]. TVAL3 often offers a fairly good solution without any specific precondition due its adaption to non-sparse sample, compared to other denoising algorithms which work best when the sample is sparse. Furthermore, the TVAL3 algorithms is preferably used due to its superiority in computational

speed. The isotropic $TV+$ model was chosen for TVAL3: $\min_{x \in \mathbb{R}^n} \sum_i \|D_i x\|_2$, s.t. $Ax = b$, $x \geq 0$; with $D_i x$ the discrete gradient of x at pixel i as described in detail in [151]. The variables can be split by introducing $y_i = D_i x$, and the TV model is now equivalent to: $\min_{y_i, x} \sum_i \|y_i\|_p$, s.t. $Ax = b$ and $D_i x = y_i$, with $p = 1$ or 2 . The following non-default parameters were set to: $\text{maxit} = 1500$, $\text{TVnorm} = 1$, $\text{nonneg} = \text{true}$ and $\text{isreal} = \text{true}$. TVAL3 allows to set an initial guess as starting reference for its reconstruction routine which might positively influence the result. In this study, the computed results of GI were used as initial guess. However, we did not notice any significant advantage of setting this parameter compared to its default setting of a zero matrix.

For comparison of CI to conventional raster scan imaging, we establish as metric the compression rate ($CR = (N \times N)/M$), which is a measure to indicated the decrease in acquisition time in CI. CR was defined by the $N \times N$ pixel size of a reconstructed image, divided by the amount of speckle patterns (M) used to illuminated the sample. A point-by-point raster scan image needs $N \times N$ acquisitions while the CI image uses only a fraction $M/(N \times N)$ but retains or even enhances resolution beyond the diffraction limit [56, 136, 138]. The higher CR, the higher the compression, the shorter the acquisition time.

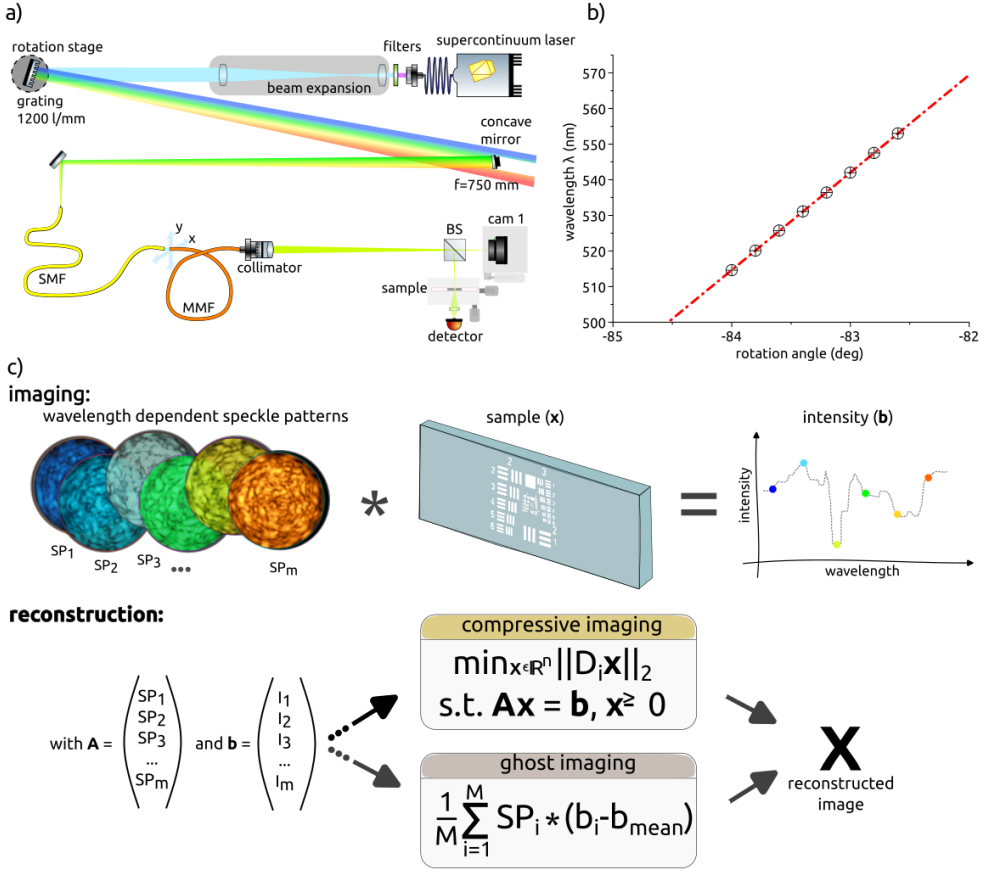


Figure 4.1: a) Schematic of the experimental setup. A diffraction grating is illuminated by the collimated VIS spectrum of a supercontinuum laser. Light, which is dispersed into separate wavelengths is coupled to a single-mode fiber, which serves as a pinhole selecting a single wavelength. The single-mode fiber is connected to the MMF. Wavelength dependent speckle patterns generated in the MMF illuminate the sample and the total response is collected by a bucket detector. b) Wavelength at the MMF output as a function of the rotation angle of the grating: experimental data (circles) and linear fit (red line). c) The main principle of swept-source compressive fiber imaging. Wavelength dependent speckle patterns “illumination matrix” $\mathbf{A}$ and the corresponding signal from the sample (vector $\mathbf{b}$) are measured and feed either into the ghost imaging algorithm or into a regularization algorithm to find the solution $\mathbf{x}$, which is the image of the sample.

4.3 RESULTS AND DISCUSSION

In the first set of experiments, different samples, like “bars”, the “zero” digit and part of a “resolution target” were imaged by the proposed swept-source fiber imaging approach. The results are shown in Figure 4.2. The wavelength scanning bandwidth was set to $\Delta\lambda = 55$ nm and 82.5 nm, and a total of $M = 1000$ and 1500 speckle patterns were used with a scanning step size of 0.055 nm. The overall acquisition time depends on the number of patterns projected onto the sample, the exposure time of the camera used to record the speckle images and the rotation stage settling time. For 1000 speckle patterns, an exposure time of 100 ms and a settling time of 2 ms, the overall imaging time was around 102 s.

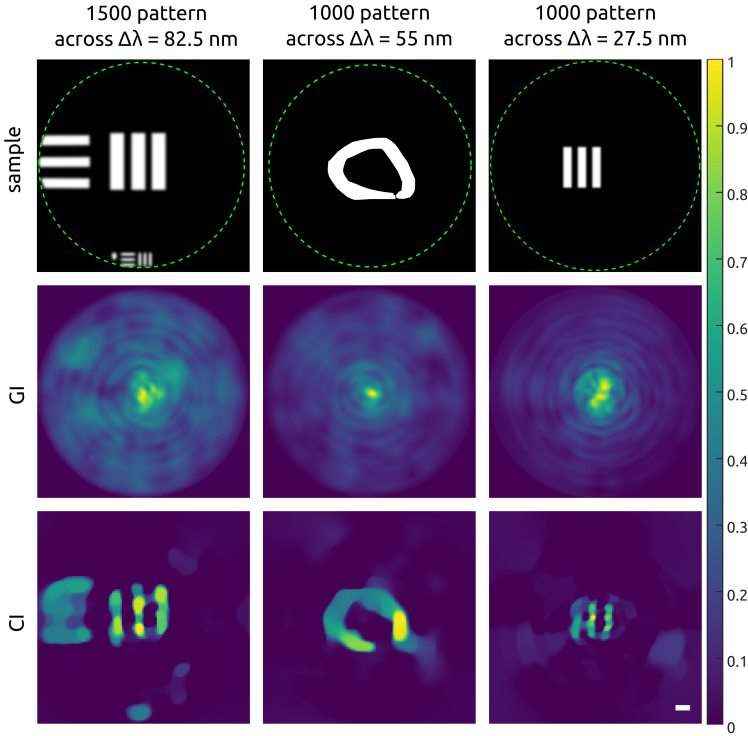


Figure 4.2: Swept-source multimode fiber imaging results for different samples (first row) using GI (middle row) and CI (bottom row) reconstruction algorithms. Resulting image size was $N^2 = 128 \times 128$. Resolution target (first column) were imaged using 1500 speckle patterns, a scanning step size of 0.055 nm and a bandwidth of $\Delta\lambda = 82.5$ nm, resulting in CR = 10.9. “Zero” sample (second column) were imaged using 1000 speckle patterns, a scanning step size of 0.055 nm and $\Delta\lambda = 55$ nm, resulting in CR = 16.4. “Three bars” sample (third column) were imaged using 1000 speckle patterns, a scanning step size of 0.027 nm and a bandwidth of $\Delta\lambda = 27.5$ nm, resulting in CR = 16.4. Scale bar: $3 \times$ diff. limit with a diff. limit of $12.6 \mu\text{m}$

The results of GI, shown in the second row of Figure 4.2, are not representing the sample but rather display an accumulation of high signals at the center of the image. This is to be expected since GI relies on a high number of pattern for good reconstruction. In contrast, the obtained CI reconstructions of the “resolution target” with $N^2 = 128 \times 128$

pxl ($555 \times 555 \mu\text{m}^2$) resembling the sample with soft and blurry edges, which allows for the sufficient identification of the imaged object. The less complex “zero” digit is rendered almost perfectly albeit some minor background artefacts are visible. Similar results were obtained for the pure “three bars” sample. Overall, most segments of the samples were well reconstructed and could be identified easily via the CI-based swept-source fiber imaging.

4.3.1 FACTORS THAT IMPACT IMAGE QUALITY

In the next set of experiments, the influence of the final image size on the reconstruction quality has been investigated. For that, the simple binary sample has been chosen as depicted in Figure 4.3a). The holes of a diameter $45 \mu\text{m}$ are separated by a center to center distance of $75 \mu\text{m}$. We opted for a scanning range of 27.5 nm with a rotation of 1° in 1000 steps, starting at 514 nm . The scanning bandwidth and the number of illumination patterns ($M = 1000$) were kept constant during the measurements.

In Figure 4.3b), the reconstructed results for different N are presented. The reconstruction image size was varied from $N = 64$ to $N = 192$, resulting in different CR from 4.1 to 36.9, respectively. The first column in Figure 4.3b) displays the GI results, which do not depend on N . The two dots are clearly visible albeit noisy artefacts are present in the background, only the pixel grid gets finer with the increasing number of pixels.

Swept-source multimode fiber imaging using CI is shown in the second column. The visibility of the two dots is increased with the number of pixels in the reconstructed image. A better signal-to-background ratio, mainly obtained by a diminishing of the background is the result of the denoising capabilities of the TVAL3 algorithm. For $N^2 = 128 \times 128 \text{ pxl}$, meaning a resolution of $4.3 \mu\text{m}$ per pixel, the holes are presented by 10 pixels, matching the size of the sample perfectly. Compared to GI, compressive imaging (with total variation denoising) increases the visibility of the two dots by dampening the background and increasing contrast by sharpening the edges of the two dots. Figure S3 in the Supplementary Information demonstrates image reconstruction quality simulated for different signal-to-noise (SNR) levels. Our simulations show that for the relatively low noise case ($\text{SNR} > 10$), good image reconstruction of a sparse sample could be achieved with the bandwidth of only 5.5 nm .

In the second set of experiments, we investigated the imaging quality as a function of bandwidth and the correspondent number of illumination patterns. The results for three different number of speckle patterns: $M = 200, 300$ and 750 , which correspond to bandwidths of $\Delta\lambda = 5.5, 8.25$ and 20.63 nm , respectively, are presented in Figure 4.3c). Since the image size of $N^2 = 128 \times 128 \text{ pxl}$ was kept constant, the CR value is changing ($\text{CR} = 81.9, 54.6$ and 21.8) based on the number of patterns used. For GI, depicted in the first column, the more patterns used the better is the result. The best results, in terms of contrast and visibility, could be obtained using CI, as shown in the second column. A bandwidth of $\Delta\lambda > 8.25 \text{ nm}$, meaning only $M = 300$ patterns ($\text{CR} = 54.6$), suffices for a convincing reconstruction of the sample. This provides an ample improvement in terms of reducing the scanning bandwidth compared to the report of Shin et al. [146] where 0.1 nm steps across 100 nm were used.

4.3.2 IMAGING VIA A FLEXIBLE PROBE

For real life applications it is crucial to maintain invariant speckle patterns when the fiber is moved. In the next set of experiments, we investigated the stability of the system over time and imaging performance while the fiber probe is moved.

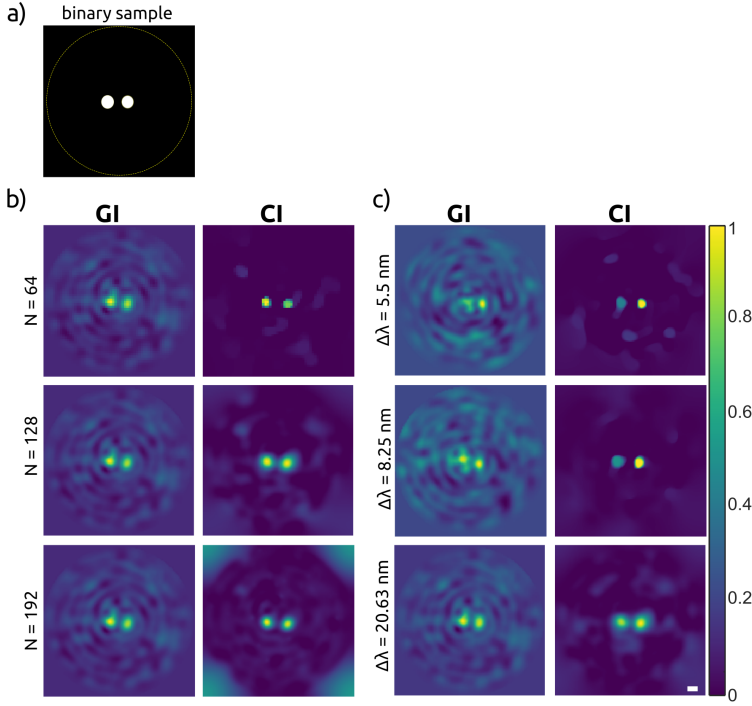


Figure 4.3: a) Sample. b) Swept-source multimode fiber imaging results for $\Delta\lambda = 27.5$ nm and $M = 1000$ patterns as a function of image size within same field of view: 64x64 pxl (top row, CR = 4.1), 128x128 pxl (middle row, CR = 16.4) and 192x192 pxl (bottom row, CR = 36.9) using ghost imaging (first column) and compressive imaging (second column) reconstruction algorithms. c) Swept-source multimode fiber imaging results for image size of $N^2 = 128 \times 128$ pxl as a function of bandwidth (or number of speckle patterns): $\Delta\lambda = 5.5$ nm (top row, $M = 200$, CR = 81.9), $\Delta\lambda = 8.25$ nm (middle row, $M = 300$, CR = 54.6) and $\Delta\lambda = 20.63$ nm (bottom row, $M = 750$, CR = 21.8) for ghost imaging (first column) and compressive imaging (second column) algorithms. For simple binary objects, imaging using a small bandwidth and high CR is feasible. Scale bar: $3 \times$ diff. limit with a diff. limit of $12.6 \mu\text{m}$

In the first experiments, we tested the reconstruction stability over time while the fiber is kept stable. The sample and the swept-source multimode fiber imaging results as a function of time in minutes are shown in Figure 4.4. At zero time, the illumination matrix $\mathbf{A}$ consisted of $M = 1000$ flattened speckle patterns in the range from $\lambda_1 = 514$ to $\lambda_N = 541.5$ nm ($\Delta\lambda = 27.5$ nm), and the correspondent signal vector $\mathbf{b}$ were measured simultaneously. The image reconstructed using the CI algorithm is presented in Figure 4.4b. It nicely represent the complex structure of the real sample. The reconstructed images have a size of $N^2 = 128 \times 128$ pxl ($2203 \times 2203 \mu\text{m}^2$). We repeated intensity measurements 5 minutes and 10 minutes later to get $\mathbf{b}_{5\text{min}}$ and $\mathbf{b}_{10\text{min}}$, respectively. Images in Fig. 4.4c) and d) show the reconstruction results for hybrid measurements: the illumination matrix $\mathbf{A}$ was measured at zero time with the assumption that the speckle patterns would not change over time and the signal from the sample was measured 5 min and 10 min later, respectively. The Pearson correlation

coefficients (corr2) between the zero time reconstructed image and images reconstructed after delayed intensity measurements are equal to 0.98 and indicated in Fig. 4.4c and d. For both time stamps, the correlation values are high, indicating an excellent agreement with the original sampling. Similar results are obtained by calculating the structural similarity index (ssim). Visually, the same assessment can be concluded. Swept-source multimode fiber imaging system provide robust measurement result over time.

In the second set of experiments, we investigated how moving and bending the single-mode part of the fiber probe would influence the imaging performance of the proposed approach. First, we measured simultaneously the illumination matrix $\mathbf{A}$ consisted of $M = 1000$ flattened speckle patterns in the range from $\lambda_1 = 514$ to $\lambda_N = 541.5$ nm ($\Delta\lambda = 27.5$ nm), and the correspondent signal vector $\mathbf{b}$. The image reconstructed using CI algorithm is presented in Fig. 4.5a). We clearly see our sample. The experimental layout of a SMF (yellow) that corresponds to the measurements is shown in Fig. 4.5d). Then, the SMF was coiled twice as shown in Fig. 4.5e) and the intensity measurements were repeated. The image presented in Fig. 4.5b) was reconstructed using the illumination matrix $\mathbf{A}$ measured for the first position of the SMF (as in Fig. 4.5d) and the intensity vector $\mathbf{b}_{2\text{loops}}$ measured for the second position of the SMF (as in Fig. 4.5e).

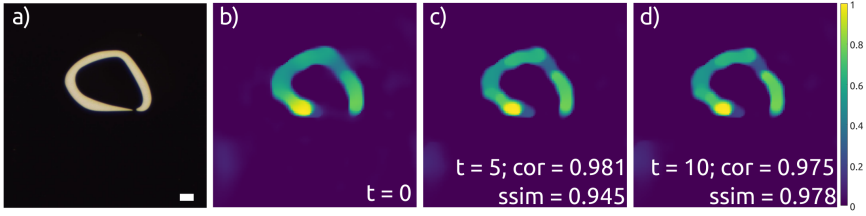


Figure 4.4: Stability over time measurements for swept-source MMF imaging using $M = 1000$ patterns over $\Delta\lambda = 27.5$ nm. (a) Sample. (b-d) CI reconstruction results for $N^2 = 128 \times 128$ and a CR of 16.4: illumination matrix $\mathbf{A}$ and intensity vector $\mathbf{b}$ were measured simultaneously (b), $\mathbf{A}$ and $\mathbf{b}_{5\text{min}}$ were measured with 5 min delay in between (c), and $\mathbf{A}$ and $\mathbf{b}_{10\text{min}}$ were measured with 5 min delay in between (d). The correlation coefficient values and structural similarity index

between (b) (c) and (b) (d) are indicated. Scale bar: $3 \times \text{diff. limit}$ with a diff. limit of $50.1 \mu\text{m}$.

Subsequently, a third loop was added, as depicted in Figure 4.5f). The image presented in Fig. 4.5c) was reconstructed using the illumination matrix $\mathbf{A}$ measured for the first position of the SMF (as in Fig. 4.5d)) and the intensity vector $\mathbf{b}_{3\text{loops}}$ measured for the third position of the SMF (as in Fig. 4.5f)). The Pearson correlation coefficients between the image presented in Fig. 4.5a) and the image presented in Fig. 4.5b) was very close to 1. The correlation coefficients between the image in Fig. 4.5a) and in Fig. 4.5c) was calculated to be 0.955. The correlation coefficient slightly drops with changing the SMF configuration from a single loop to three loops. Nevertheless, taking the harsh treatment and bending into account, the overall image reconstruction quality remains very good. The “zero” can be clearly distinguished from the background, although a minute increase in background noise can be noted in Fig. 4.5c). We conclude, that the deformation of the SMF has no major influence on the reconstruction quality even for quite extreme bending. However, it is important not to lose sight of the fact that the one meter long MMF remains the sensitive part of the setup. External

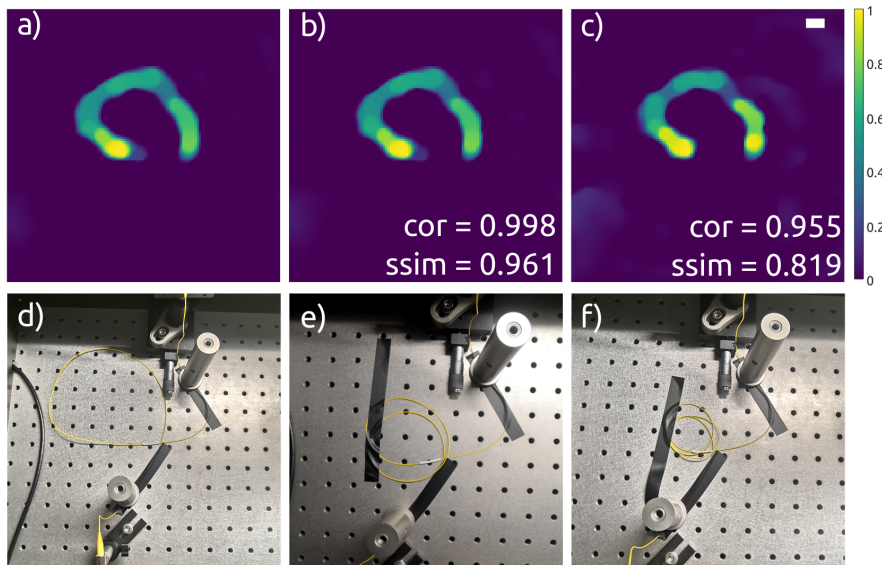


Figure 4.5: Stability measurements of the proposed swept-source MMF imaging ($\Delta\lambda = 27.5 \text{ nm}$, $M = 1000$) during fiber movements. Image reconstruction results (a-c) and the correspondent configuration of the SMF (d-e). Pre-calibration has been done only once. Scale bar: $3 \times \text{diff. limit}$ with a diff. limit of $50.1 \mu\text{m}$

influence like bending or movement will alter the light transmission through the MMF and would constantly require new calibration. To establish a robust but flexible endoscopic fiber imaging system, a shorter MMF part should be used. In addition, further ideas to generate invariant and repetitive speckle patterns (e.g., 3D printed diffractive optical elements or meta-material layers combined with swept-source illumination) could be explored.

4.4 CONCLUSION

In this study, we proposed and demonstrated a swept-source fiber imaging approach. We have shown the feasibility of using wavelength dependent fiber-generated speckle patterns for fiber-based compressive imaging. We were able to reconstruct a variety of samples using a bandwidth of $\Delta\lambda < 10 \text{ nm}$ in the visible range. The relatively narrow VIS bandwidth could potentially be used for fluorescence imaging applications. The proposed imaging approach does not require any mechanical scanning device, which simplifies the whole illumination process compared to experimental setups with galvanometric mirrors or DMDs [88, 139]. Surely, the in-house-built monochromator is an over-engineered device, and can be replaced by a supercontinuum laser with an acousto-optic tunable filter or by a rapid swept source laser as commonly used in optical coherence tomography or by a tunable laser as reported lately [146, 147, 152]. We experimentally demonstrated that the imaging quality of the proposed approach doesn't require stability of the fiber probe. The results open up new avenues in the field of label-free and label-based deep tissue imaging, potentially enabling *in vivo* all-fiber based endoscopic high resolution imaging.

SUPPLEMENTARY INFORMATION

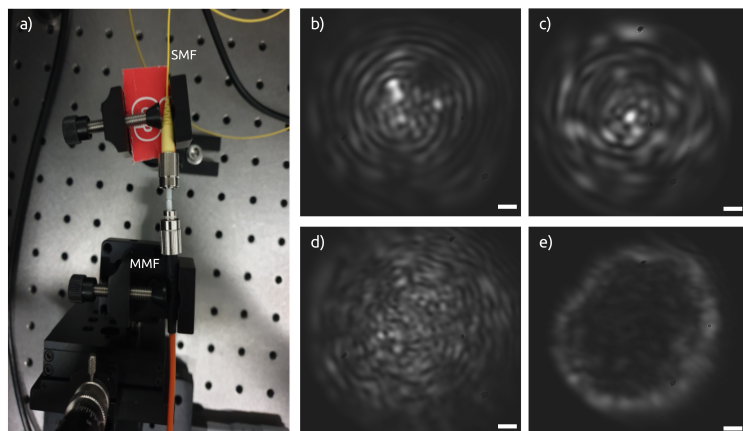


Figure S1: a) Free space fiber coupling between the single-mode fiber (SMF, Thorlabs P1-460Y-FC-1) and the multimode fiber (MMF, Thorlabs M42L01). b-e) obtained speckle patterns for $\lambda = 487.29 \text{ nm}$ while changing coupling position. b) and c) coupling close to the MMF center. d) Coupling slightly more off-axis, generating a homogeneous speckle pattern (preferred configuration) and d) coupling close to the core edge and hence guiding partially in the cladding. Scale bar: $200 \mu\text{m}$.

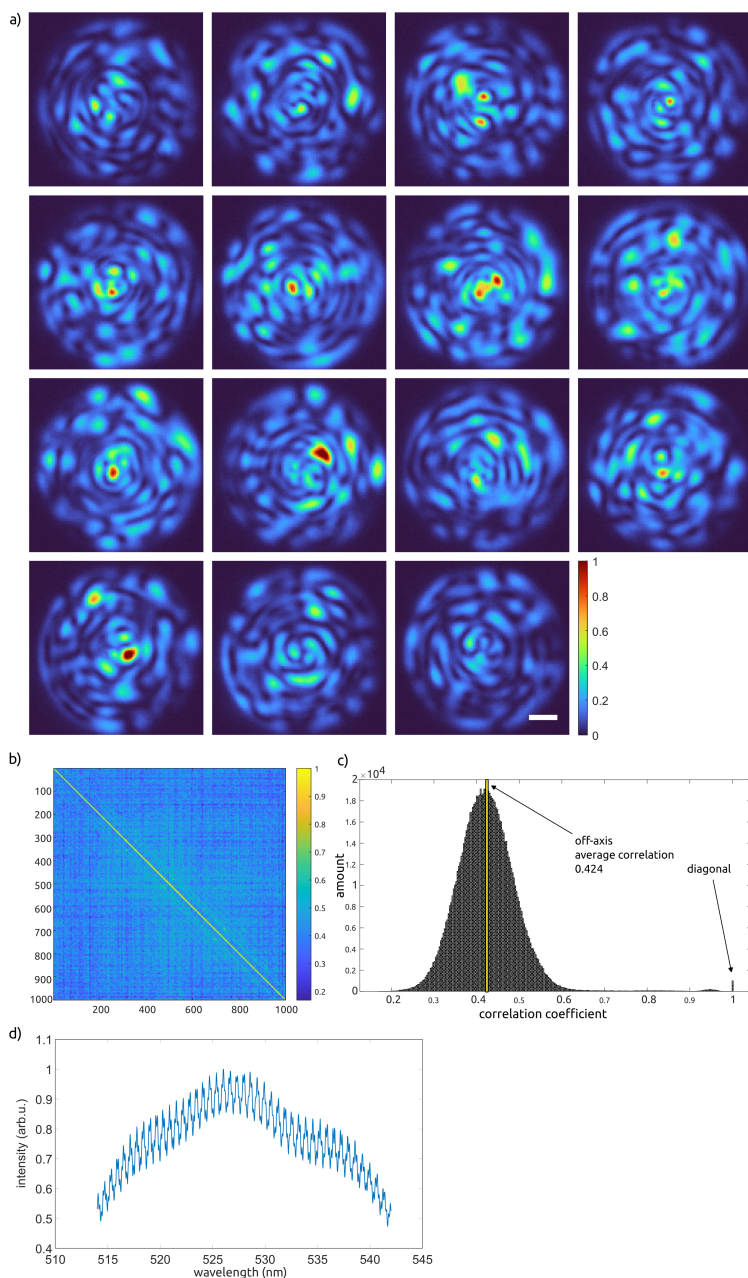


Figure S2: a) Examples of normalized speckle pattern for 15 different wavelengths between 514 to 541.5 nm in steps of 1.83 nm. The scale bar is 100 μm . b) Correlation graph and its corresponding c) correlation histogram calculated for every pair of experimentally measured speckle patterns. The off-axis average correlation of 0.42 (orange line). Please note that the off-axis correlation value is higher than expected due to the zero padding in the corners (of the circularly cropped speckle patterns). d) Normalized speckle intensity as a function of wavelength.

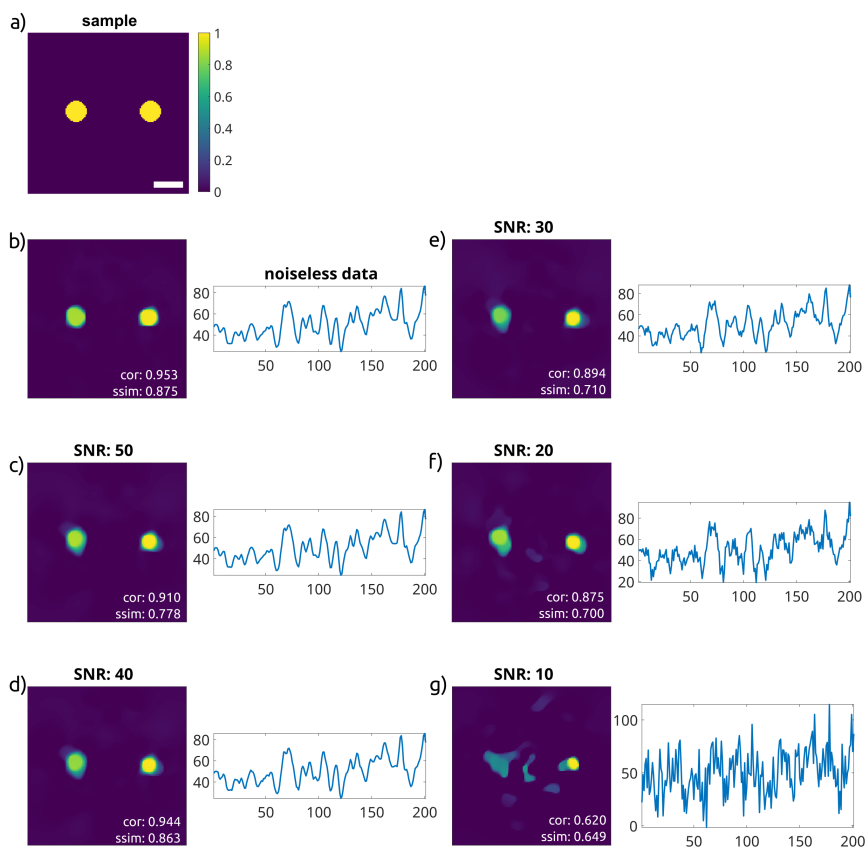


Figure S3: Swept-source fiber imaging simulation results. A set of 200 experimentally pre-recorded speckles patterns with a total scanned bandwidth of 5.5 nm is used as illumination matrix **A**. (a) The binary sample. (b-g) Reconstructed images and the simulated data (the vector **b**) for different levels of noise. The Pearson correlation coefficient and the structural similarity index relative to the original sample are indicated within each image. The diffraction limit is 12.6 μm . The scale bar is 100 μm .

Swept-source fiber imaging with different noise levels was simulated as presented in Figure S3. For the simulations, we used 200 experimentally recorded speckle patterns with a total scanning bandwidth of only 5.5 nm (a step size of 0.0275 nm) and binary sample object $\mathbf{x}$ (two dots presented in Fig. S3(a)). Each speckle has a size of $N^2 = 128 \times 128$ pxl (pixel size is $4.3 \mu\text{m}$) corresponding to $555 \times 555 \mu\text{m}$ field of view. The optical diffraction limit is $12.6 \mu\text{m}$. The two dots have a diameter of 16 pxls each, which equals to $68.8 \mu\text{m}$.

The speckle patterns (each pattern corresponds to a row in matrix $\mathbf{A}$) were used to “illuminate” sample $\mathbf{x}$. Illumination was simulated by the multiplication of each speckle pattern with the sample pixel-wise. The total response, which is one element in the vector $\mathbf{b}$, was calculated by integration. The regularization algorithm, TVAL3, was fed by the illumination matrix $\mathbf{A}$ and the signal vector $\mathbf{b}$ to reconstruct the sample. The results for noiseless simulations are presented in Fig. S3(b).

To simulate the reconstruction under different signal-to-noise (SNR) conditions, we added white Gaussian noise to the simulated signal $\mathbf{b}$ by using the `awgn` function (MATLAB).

The SNR was defined as the difference in variance, $\Delta\sigma$, between the noiseless data $\mathbf{b}$ and generated noisy signal, $\mathbf{b}_{SNR}$.

5

QUANTITATIVE PHASE IMAGING WITH A MULTIMODE FIBER

5

Label-free quantitative phase imaging is vital for optical microscopy and metrology applications. A multimode fiber stands out as a desirable platform for imaging. Here we propose and experimentally demonstrate a non-interferometric non-iterative approach for high-speed high-resolution label-free quantitative phase imaging via a random light scattering in a multimode fiber.

5.1 INTRODUCTION

MMFs have recently emerged as an ultimate endoscopic technology that enables high-resolution imaging at the tip of a hair-thin flexible probe [153, 154]. A wide range of imaging modalities through MMF-based endoscopes have been demonstrated, including (auto-)fluorescence [76, 155–157], confocal reflectance [158, 159], chemically-selective [160], super-resolution [161, 162], and 3D [163, 164] imaging. However, even nowadays, state-of-the-art fiber-based microscopy relies on amplitude-only contrasts. While to visualize ‘phase objects’, such as refractive index variations or matter density, quantitative phase information is required [90, 165, 166]. As conventional detectors record only the amplitude of light, advanced approaches are needed to recover the phase. This fundamental problem stimulates the development of many techniques. Quantitative phase imaging (QPI) is a set of label-free methods to measure the optical path length delays within the sample. The problem of recovering the complex wavefront can be solved by interferometric approaches. By recording interference between an unknown field and a known reference, the incident field can be reconstructed. The spatial resolution of holographic phase microscopy can be improved by speckle field illumination [167]. Holography requires extreme environmental stability and a high-quality reference beam limiting their realizations and applications [168]. A broad range of techniques that eliminates the use of a reference include Fourier ptychography [169] and differential phase contrast microscopy [170]. These methods reconstruct the phase shift from a relatively large sequence of intensity patterns and require iterative computationally expensive post-processing. One of the most popular non-interferometric QPI methods is Shack–Hartmann (SH) wavefront sensing [171]. A lenslet array creates focused spots, the positions of which depend linearly on the local phase. A similar principle of using a microlens array for multiplexing the angular domain into the spatial domain is also broadly used in light-field cameras [172]. However, the lenslet manufacturing parameters – total number, focal length, curvature, and diameter of lenses – limit the resolution and sensibility. Multiple solutions that utilize structurally modulated light have been proposed to overcome the limitations [173–176]. Soldevila et al. [176] demonstrated QPI using a sequence of binary amplitude-only patterns and a single focusing lens. However, this imposes strict limits on the illumination system.

Despite all the recent progress, the development of a compact high-resolution QPI remains an important goal. Recently, a reference-free holographic image sensor, which exploits a commercial diffuser as a holographic lens has been proposed [177]. Introduction of fiber requires complex probe designs: QPI through a fiber bundle with a GRIN lens [178] and multicore fiber has been demonstrated [179, 180]. These methods require capturing several high-resolution images, and/or complex iterative or machine learning-based reconstruction. High spatial resolution is another major challenge of QPI, especially for compact non-interferometric sensors [181, 182]. The lateral resolution of optical imaging is generally limited by the diffraction of light:

$$\mu_{min} = k \frac{\lambda}{NA}, \quad (5.1)$$

where μ_{min} is the smallest resolvable distance, λ is the wavelength of the incident radiation and NA is the numerical aperture of the optical system. The critical parameter k is determined by the imaging modality and yields either 0.5 or 0.82 for conventional incoherent and coherent

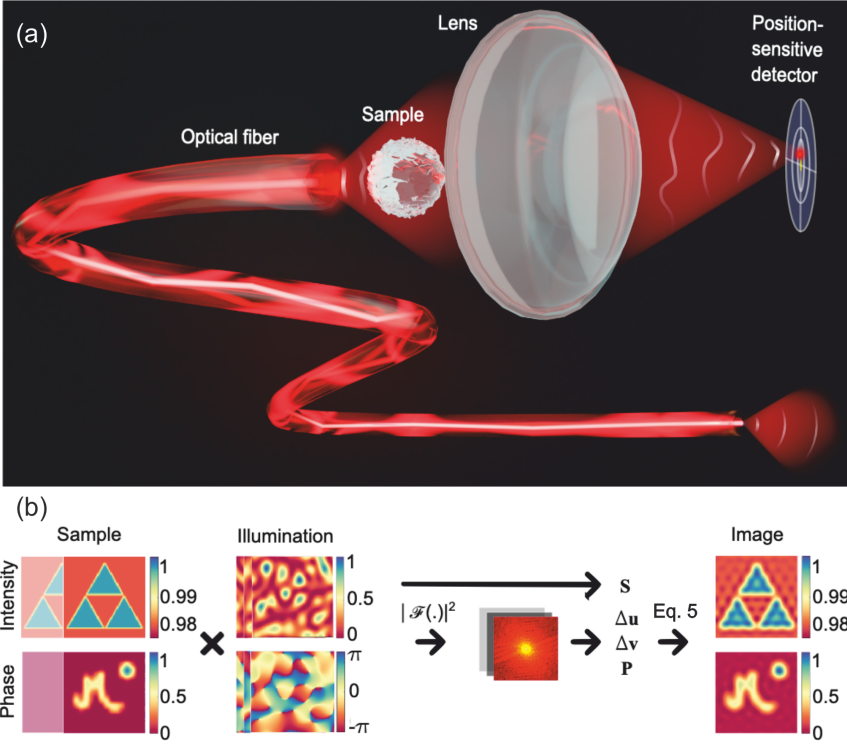


Figure 5.1: (a) Schematic of the experimental setup. (b) Experimental workflow depicted through the simulation results. Complex speckle fields (second column) illuminate the unknown sample (first column). The center-of-mass coordinates in the Fourier plane relative to the “reference” sample, Δu_n and Δv_n , together with the total transmitted power P_n , are recorded by the position-sensitive detector. The Fourier transform is denoted by $\mathcal{F}$, and $|\mathcal{F}(\cdot)|$ indicates its magnitude. The matrix $\mathbf{S}$ is measured during pre-calibration. The sample phase and amplitude are reconstructed using the proposed framework.

imaging, respectively. Therefore, the lateral resolution of QPI is generally determined by the $0.82\lambda/NA$ diffraction limit [182, 183].

We propose and experimentally demonstrate non-inferometric quantitative phase imaging using random speckle illumination of an MMF. To the best of our knowledge, this is the first demonstration of QPI utilizing centroid coordinates and random complex illumination. An appealing feature is that the lateral resolution is limited by the incoherent diffraction limit of the illumination, which is 1.5 times better than the diffraction limit of conventional QPI. Moreover, we eliminate the need for a camera and rely solely on position-sensitive detection (PSD). We experimentally demonstrate quantitative phase reconstruction of a variety of phase targets with a simple noniterative algorithm.

5.2 MAIN PRINCIPLE

The main idea of the proposed approach is illustrated in Fig. 5.1(a). Coherent light coupled into an MMF gives rise to a complex speckle pattern, $B_m(x, y)e^{i\psi_m(x, y)}$. We use a set of these seemingly random patterns to sample the unknown wavefront. Each pattern illuminates the unknown sample, $A(x, y)e^{i\phi(x, y)}$ and a single lens is used to form phase-sensitive intensity distributions in the Fourier plane for transmitted or reflected light – a principle also fundamental to Shack–Hartmann sensors or light-field cameras. A position-sensitive detector in the back focal plane of the lens records the coordinates of the center of mass, (u_m, v_m) , and the total transmitted intensity, P_m . Both, the amplitude and phase of the sample can be computationally recovered using the linear relationship between the local wavefront slope at the sample plane and centroid displacement in a Fourier plane [184, 185]. As we derive in Appendix A, the position of the centroid (u_m, v_m) is linked to the gradient of the phase by:

$$\begin{pmatrix} u_m \\ v_m \end{pmatrix} = \frac{\lambda F}{2\pi} \frac{1}{P_m} \iint (B_m(x, y)A(x, y))^2 \nabla (\phi(x, y) + \psi_m(x, y)) dx dy, \quad (5.2)$$

5

where F is the focal distance of the lens; $A(x, y) \leq 1$ is the amplitude attenuation of the sample; $\phi(x, y)$ is the phase delay introduced by the sample; $B_m(x, y)$ is the amplitude and $\psi_m(x, y)$ is the phase of the m -th speckle pattern; $P_m = \iint B_m^2(x, y)A^2(x, y) dx dy$ is the total power transmitted through the sample for each speckle pattern and $\nabla = (\partial/\partial_x, \partial/\partial_y)$ is the gradient in two dimensions.

The main challenges of solving Eq. (5.2) for unknown $\phi(x, y)$ and $A(x, y)$ include the computational reconstruction of the wavefront at all points in space from ‘single-pixel’ measurements (centroid position), demultiplexing, and the distillation of the sample field from the field of complex illumination patterns. As derived in Appendices B and C.1, the problem can be solved directly if the full calibration of the imaging system is performed and both amplitude $B_m(x, y)$ and phase $\psi_m(x, y)$ of illuminating speckle patterns are known. While this full transmission matrix pre-calibration is experimentally possible [186], in real-life scenarios, it might be difficult to measure phase profiles of the speckle patterns.

We propose an elegant solution to reconstruct the unknown sample wavefront using simplified pre-calibration – only intensity maps $S_m(x, y) = B_m^2(x, y)$ of the speckle patterns. While this approach still requires pre-calibration, the recording of intensity patterns without phase stepping or external references is simpler than the full transmission matrix measurements. The proposed approach works for phase-only samples as well as for measuring (e.g. time-dependent) phase changes of a sample with a constant amplitude profile. The key is to implement a differential measurement scheme and use a set of centroid positions (u_{m0}, v_{m0}) of a reference, e.g. flat phase, sample. For continuous monitoring of phase changes of a given sample, (u_{m0}, v_{m0}) are the coordinates of the centroid position at a previous time-stamp. The gradient of the sample phase can be reconstructed without knowing the complex phase distributions of illuminating speckle patterns, as derived in Appendix C.2.

The unknown amplitude and phase can be represented by vectors $\mathbf{a} \in \mathbb{R}^N$, $\partial\phi/\partial_x \equiv \phi_x \in \mathbb{R}^N$, $\partial\phi/\partial_y \equiv \phi_y \in \mathbb{R}^N$, which are the squared amplitudes $A(x, y)^2$ and phase gradient $\nabla\phi(x, y)$ on a grid of N pixels. Similarly, we represent the measured relative centroid positions in the x and y directions corresponding to the M speckle patterns by $\Delta\mathbf{u} \in \mathbb{R}^M$, $\Delta\mathbf{v} \in \mathbb{R}^M$. The unknown amplitude and phase from these measurements can be reconstructed

as follows:

$$\mathbf{a} = \mathbf{S}^\dagger \mathbf{p},$$

$$\begin{pmatrix} \phi_x \\ \phi_y \end{pmatrix} = \begin{pmatrix} \mathbf{M}^\dagger & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^\dagger \end{pmatrix} \begin{pmatrix} \Delta \mathbf{u} \\ \Delta \mathbf{v} \end{pmatrix}, \quad (5.3)$$

where

$$\mathbf{M} = \frac{\lambda F}{2\pi} \text{diag}(\mathbf{p})^{-1} \mathbf{S} \text{diag}(\mathbf{a}), \quad (5.4)$$

with $\mathbf{S} \in \mathbb{R}^{M \times N}$ consisting of M rows of $\text{vec}(B_m(x, y)^2)$, and $\mathbf{p} \in \mathbb{R}^M$ a vector with the total transmitted power, P_m , for each speckle pattern. The operator $\text{diag}(\cdot)$ constructs a matrix with the given vector on its diagonal, the operator $\text{vec}(\cdot)$ converts the matrix into a vector, and $\cdot^\dagger$ denotes the Moore-Penrose pseudo-inverse [187]. We can explicitly compute the pseudo-inverse through the singular value decomposition (SVD), and regularise the computation by truncating small singular values. Note that in case $M < N$, the pseudo-inverse can only recover up to elements in the null-space of the matrix. The amplitude map $A(x, y)$ is reconstructed by reshaping and calculating the square root of $\mathbf{a}$ and the quantitative phase map $\phi(x, y)$ is calculated by reshaping and numerically integrating ϕ_x, ϕ_y .

5

5.3 SIMULATIONS

The experimental workflow is schematically shown in Fig. 5.1(b) where we also present our simulation results. We simulate the entire procedure of phase and amplitude imaging via the proposed approach. A complex sample ($N = 92 \times 92$ pixels) is presented in Fig. 5.1(b, first column) together with a ‘reference’ wavefront. As an illumination, a set of $M = 92 \times 92 = 8464$ fully-developed random speckle patterns with the diffraction-limited speckle size is generated. We assume the speckle patterns to be known or pre-characterized experimentally. Examples of illumination phase $\psi_m(x, y)$ and intensity $S_m(x, y)$ distributions are presented in Fig. 5.1(b, second column). The finite NA of an MMF has been simulated by using a circular coherent transfer function in the Fourier plane [188]. Each speckle pattern illuminates the sample, or, in simulations, is multiplied by a sample complex transmission function $A(x, y)e^{i\phi(x, y)}$. The intensity distributions in the Fourier plane are measured at the back focal plane of the lens or simulated by the Fourier transform [184]. The total transmitted power and the centroid coordinates for each speckle illumination pattern are obtained. The procedure is repeated for the reference object with a plane phase profile. The amplitude of the sample is reconstructed using the first expression in Eq. 5.3, which follows a similar approach to speckle-based compressive imaging through the MMF [154]. Phase gradients along the x - and y -directions are computed using the second expression in Eq. 5.3. The spatial phase profile $\phi(x, y)$ is reconstructed by numerical integration. The results of the simulations and reconstructed amplitude and phase distributions are shown in Fig. 5.1(b, right).

The proposed MMF-based QPI can be seen as a generalized ultra-compact version of a SH phase sensor. The spatial resolution of the SH type of sensors is limited by the lenslet size [189] or a pattern aperture size [176], which in our case is equal to the average speckle size of $0.5\lambda/NA$. Therefore, the resolution of MMF-based QPI is expected to be higher than the resolution of conventional, e.g. holographic, coherent phase imaging techniques, which are limited by $0.82\lambda/NA$. To characterize the lateral resolution of the proposed approach, we

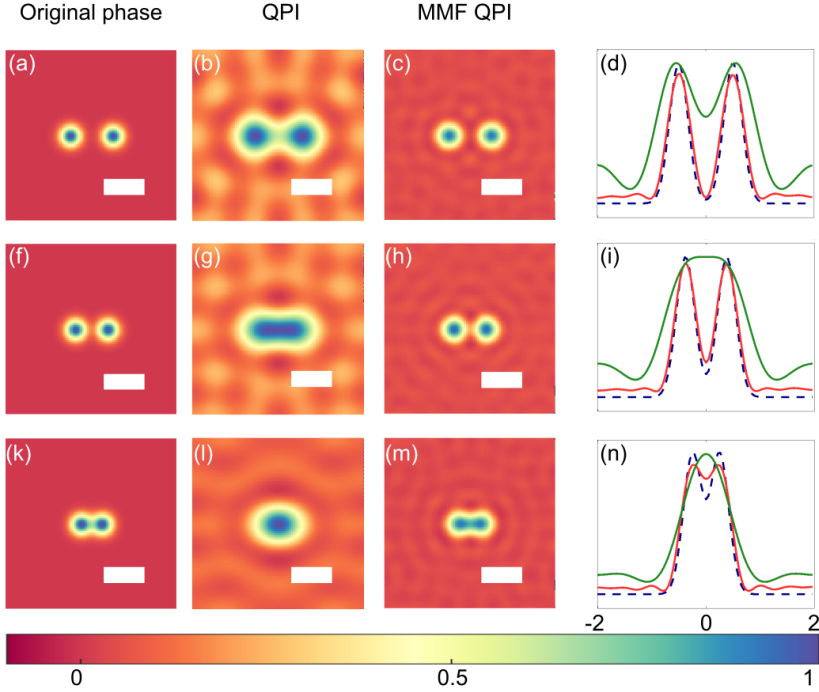


Figure 5.2: Simulation results. (a,f,k) Phase samples with distances between the dots of λ/NA (a), $0.76\lambda/NA$ (f), and $0.5\lambda/NA$ (k). Imaging results for conventional coherent imaging (b,g,l) and proposed MMF imaging (c,h,m). Scale bars are $0.82\lambda/NA$. (d,i,n) Horizontal cross-sections along the peaks for the ground-truth phase (blue dashed lines), conventional imaging (green solid lines), and MMF imaging (red solid lines). The x-axis is normalized to λ/NA .

simulate imaging of phase-only samples consisting of two phase objects separated by different distances: λ/NA , $0.76\lambda/NA$, and $0.5\lambda/NA$, as depicted in Figure 5.2(a,f,k), respectively. To simulate conventional coherent diffraction-limited imaging without the loss of generality, we assume that the phase sample is illuminated with a plane wave and account for diffraction effects by applying the low-pass circular coherent transfer function with the cutoff frequency NA/λ in the Fourier plane. The procedure is detailed in Supplementary 2 and follows the standard coherent imaging formulation described by Goodman [190]. The MMF-based QPI has been simulated as described above (see Fig. 5.1(b)) using a set of $92 \times 92 = 8464$ fully-developed random speckle patterns. We ensure diffraction-limited illumination by cutting high spatial frequencies of simulated speckle patterns in the Fourier plane. The results are presented in Fig. 5.2(b,g,l) for conventional phase imaging and in Fig. 5.2(c,h,m) for the proposed MMF-based QPI. The horizontal cross-sections along the centers of peaks are shown in Fig. 5.2(d,i,n) by the solid green line for the conventional and by the solid red line for the proposed approach, respectively. The ground truth is shown by the dashed blue line. As expected, the phase structures are distinguishable in Fig. 5.2(b) and not resolved in Fig. 5.2(g and l), where the feature size is smaller than $0.82\lambda/NA$. In contrast, the proposed

speckle-based QPI allows to resolve peaks at shorter distances as can be seen in Figure 5.2(h, m) up to a limit of $0.5\lambda/NA$. Our simulations confirm that the proposed workflow can be used for MMF-based QPI. .

5.4 EXPERIMENTAL RESULTS

We experimentally demonstrate the MMF-based QPI using a standard MMF (Thorlabs, FG050LGA) with $NA_{\text{fiber}} = 0.22$, core diameter of $50\text{ }\mu\text{m}$ and a length of about 20 cm, which is illuminated by a coherent laser beam with a wavelength of $\lambda = 640\text{ nm}$. To generate various speckle patterns, we scan the input facet with a focused laser beam on a circular uniform grid by displaying different amplitude gratings on the digital micromirror device, DMD (VIALUX, V9001VIS, 2560×1600 micromirrors with $7.6\text{ }\mu\text{m}$ pixel pitch). The DMD is located in the Fourier plane of the input objective (Olympus Plan Achromat, $20\times$, $WD = 1.2\text{ mm}$, $NA = 0.4$, $f_{\text{objl}} = 9\text{ mm}$).

The spatial frequency of the projected grating was sampled uniformly $[k_x, k_y] = -k_0..k_0$ with 20 steps, which results in up to 400 incident patterns.

As a versatile re-configurable phase sample we use a liquid crystal spatial light modulator, SLM, (Meadowlark Optics, E19x12-350-850-HDM8, 60Hz frame rate, 1920×1200 , pxl size $8\text{ }\mu\text{m}$). Speckle patterns from the MMF output are magnified $63\times$ to match the screen size of the SML. As a result, the illumination NA is $NA_{\text{fiber}}/63 = 3.5 \cdot 10^{-3}$. The field of view is restricted to $780\text{ }\mu\text{m}$ by $780\text{ }\mu\text{m}$ or 90×90 SLM pixels using a $500\text{ }\mu\text{m}$ square pinhole PH2 (Thorlabs, S500QK) in an intermediate image plane.

The intensity distributions of the speckle patterns, $S_m(x, y) = B_m(x, y)^2$, are recorded by a CMOS camera (IDS, 118 fps, 10-bits, exposure time 0.05 ms) during the pre-calibration step that is taking less than 4 s for 20×20 patterns. Additionally, about 2 sec was required to transfer data to the SSD. Images are down-scaled to $N = 120 \times 120$ pixels. Therefore, the size of matrix **S** in our experiments is $M \times 14400$. The examples of illumination patterns are presented in Fig. 5.3(a). By investigating the averaged frequency content of the patterns, as presented in Fig. 5.3(b), we confirm that the average speckle intensity size is $0.5\lambda/NA$. The diffraction limit together with the limited field-of-view put a threshold on the total number of uncorrelated speckle patterns. Our experimental configuration is expected to generate approximately 100 independent speckle patterns. To estimate the number of independent speckle patterns, we calculate the rank of the experimentally measured matrix **S** by a singular value decomposition. The singular values are presented in Fig. 5.3(c) and the rank of **S** matches with the number of independent speckle patterns. We test the alignments of the setup by imaging simple phase samples consisting of linear phase gradients in 2D. The detailed setup and alignment experiments are discussed in Supplementary 1.

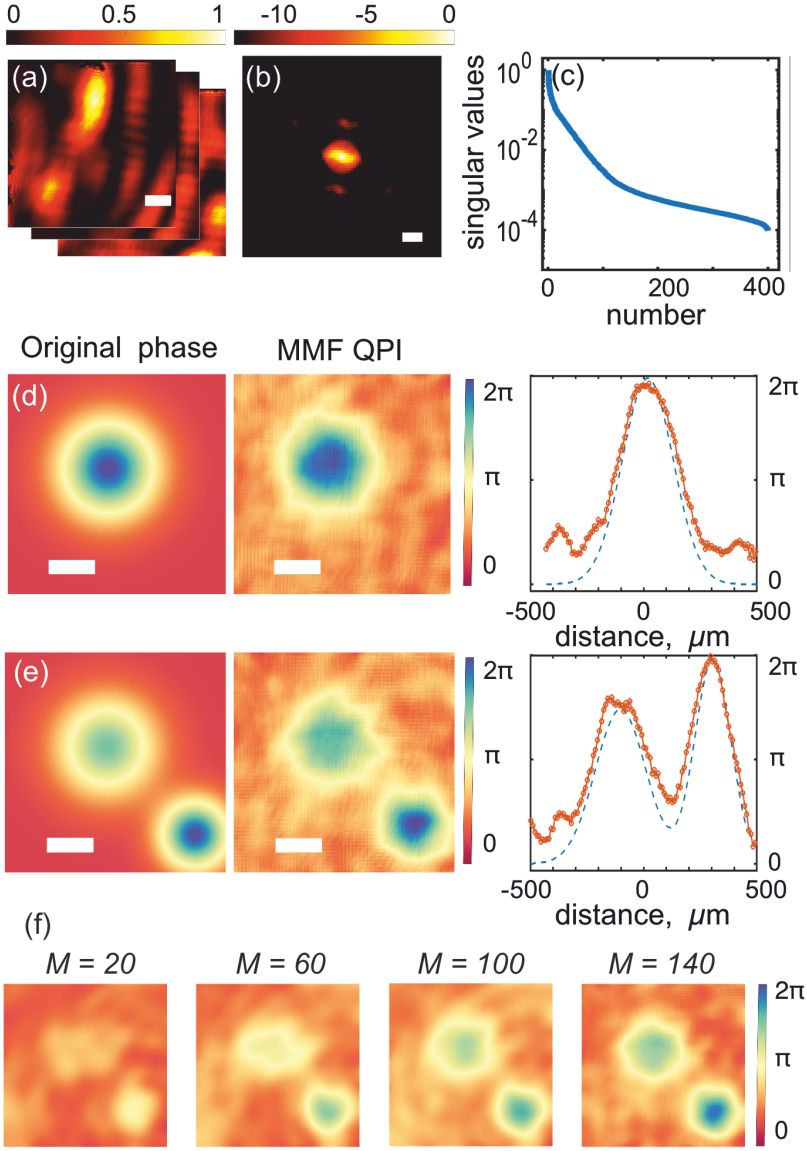


Figure 5.3: Experimental results. (a) Speckle patterns generated by the MMF, $S_m(x, y)$. The scale bar is $0.5\lambda/\text{NA} \approx 92 \mu\text{m}$. (b) Zoomed-in average power spectrum density of the speckle patterns. The scale bar is $2\text{NA}/\lambda$. (c) Singular values of $\mathbf{S}$ plotted in the descending order. (d,e) MMF-based phase imaging results: phase patterns on the SLM (left), experimentally measured and reconstructed phase (middle), and the cross-sections along the center of the peaks (right) of the original patterns (dashed blue line) and their images (solid orange line). The scale bars are $0.82\lambda/\text{NA} = 150 \mu\text{m}$. (f) Imaging quality as a function of the number of illumination patterns, M , for the sample presented in (e).

In the next set of experiments, we image different Gaussian phase distributions projected

on the SLM. As an illumination, $M = 200$ random speckle patterns on the output of an MMF have been used. The intensity distribution of each speckle pattern $S_m(x, y)$ and the corresponding centroid coordinates for the flat phase profile, u_{m0}, v_{m0} , have been measured during pre-calibration. In the imaging step, the same set of speckle patterns has been used to illuminate the phase sample projected on the SLM and the total power and the centroid coordinates have been recorded. Then $\Delta u = u_m - u_{m0}$, and $\Delta v = v_m - v_{m0}$ have been calculated. The simplified version of equation (5.3), where $A = 1$ has been used to calculate phase gradients:

$$\nabla\phi = \frac{2\pi}{\lambda F} \mathbf{S}^\dagger \text{diag}(\mathbf{P}) \begin{pmatrix} \Delta \mathbf{u} \\ \Delta \mathbf{v} \end{pmatrix}. \quad (5.5)$$

Resulting $\nabla\phi_x$ and $\nabla\phi_y$ have been reshaped to 2D matrixes of 120×120 pixels each. Phase has been calculated using inverse (integrated) gradient matlab code.

The reconstruction results are presented in Fig. 5.3(d,e). We see an excellent quantitative match between the reconstructed images and the original phase patterns. The cross-section along the centers of Gaussian peaks of the projected patterns (dashed blue lines) and phase images (orange lines) agree. To investigate how the image quality scales with the number of speckle illumination patterns, we performed phase reconstruction using various sets of illumination patterns. The experimental results for $M = 20$, $M = 60$, $M = 100$, and $M = 140$ speckle patterns are shown in Fig. 5.3(f) from left to right. Accurate shapes begin to emerge with M as small as 60 patterns, although the image still exhibits strong noise and low contrast. A clear improvement is observed with larger values of M . For $M = 140$ the image is already quite good with a relatively smooth background, while for $M = 200$ the experimentally measured images contain a grid-like structure and an oscillatory background. This can be explained by several main factors, apart from evident experimental noise. The reconstruction algorithm uses hard thresholding of singular values, which can be seen as a generalization of Fourier low-pass filtering. This kind of background is fundamental to coherent imaging techniques and can also be seen in simulated noise-free images both for conventional and proposed approaches (Fig. 5.2). In addition, matrix $\mathbf{S}$ is recorded in reflection from the SLM, which results in the superposition of the SLM pixels with the incident speckle patterns. The periodic pixel boundaries are becoming visible in the final high-resolution images.

In the third set of experiments, we image different complex phase distributions using $M = 400$ speckle patterns generated in an MMF, center-of-mass position detection scheme, and amplitude-only pre-calibration approach. The phase imaging results for complex phase samples representing letter 'O' (Fig. 5.4(a)), university logo 'VU' (Fig. 5.4(b)), and digit '9' (Fig. 5.4(c)) are presented in Fig. 5.4(d-f), respectively. Finally, to experimentally investigate the resolution limits of the proposed MMF QPI, we image the sample consisting of two Gaussian phase profiles (see Fig. 5.4(g), bottom) at a close distance of $92 \mu\text{m}$, which matches the theoretical resolution limit of the proposed MMF-based QPI calculated as $0.5\lambda/\text{NA}$. The experimental results are presented in Fig. 5.4(g), top. The cross-sections along the peaks shown in Fig. 5.4(h) demonstrate the excellent agreement between the original phase sample (dashed blue line) and the reconstructed image (solid red line), confirm the theoretically predicted resolution enhancement.

To summarize, we propose and experimentally demonstrate MMF QPI based on random complex speckle illumination and center-of-mass position detection. We have developed the necessary mathematical framework, conducted simulations, and completed proof-of-

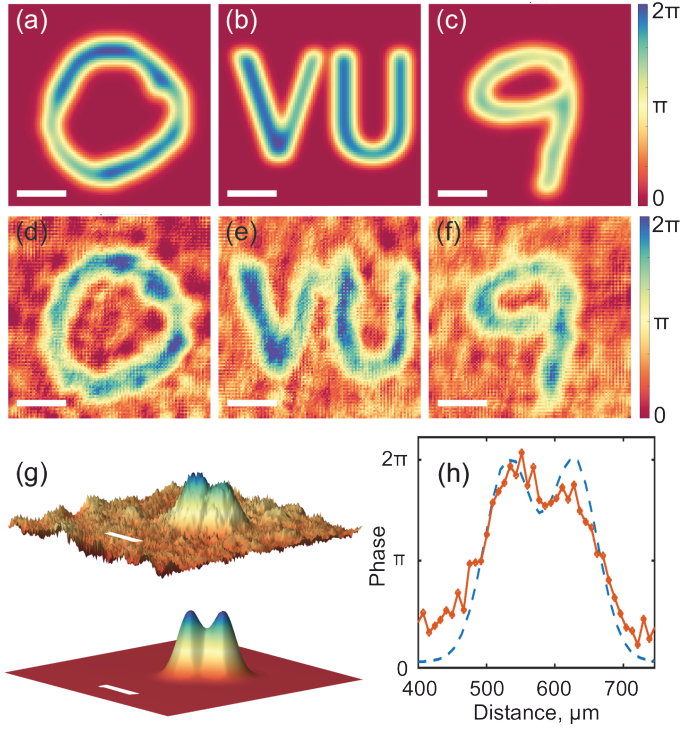


Figure 5.4: MMF-based QPI of complex phase targets. (a-c) Sample phase patterns projected onto the SLM. (d-f) Experimentally measured phase patterns representing the letter ‘O’ (d), the university logo ‘VU’ (e), and digit ‘9’ (f). (g,h) Experimental demonstration of high spatial resolution: (g, top) image of a phase pattern consisting of two Gaussian profiles at a close distance of $0.5\lambda/NA$ and (g, bottom) the original SLM pattern. The scale bars are $0.82\lambda/NA = 150 \mu m$. (h) Cross-sections along the centers of Gaussian peaks for the SLM pattern (the dashed blue line) and MMF-based image (the orange line).

principle reflection mode experimental validation. The proposed approach can be generalized to arbitrary complex patterns and further utilized in various imaging configurations. The simulations and experimental results reveal about 1.5 times resolution enhancement compared to conventional coherent imaging. Note that the proposed approach does not require reference illumination, iterative reconstruction, any additional assumptions, or free variables. The spatial resolution and imaging speed can be further improved by advanced computational approaches, which take advantage of prior knowledge and regularization.

While the research on the ultimate phase sensitivity and maximum imaging speed lies beyond the scope of this proof-of-principle study, we outline and discuss the main timing constraints.

The camera is only required for the pre-calibration step, which, while relatively time-consuming, is performed once before the imaging experiments. During phase imaging, only the center-of-mass position needs to be detected, which can be achieved by a high-speed PSD. Consequently, the imaging speed is fundamentally limited by the scanning rate and the total number of measurements. Commercial DMDs can provide a scanning rate of up to

22 kHz. Our experiments show that for the given FOV, 400 measurements and an exposure time of 0.05 ms lead to a good phase reconstruction. Thus, the proposed approach has the potential to operate at video-rate speeds, reaching 50 frames per second. As it doesn't require iterative optimization, the computational run-time does not limit the imaging speed. The most time-consuming computational step is the matrix inversion ($\mathbf{S}^\dagger$ in Eq. 2.5), which can be pre-computed in advance as a part of the pre-calibration procedure. Therefore, our approach offers relatively high-speed performance compared to other scanning techniques (e.g. ptychography), while single-shot interferometric methods operating at camera frame rates (such as off-axis holography) are inherently faster.

Although we rely on a precise calibration of an MMF, calibration does not require interferometric measurements and is therefore much simpler than measuring the full transmission matrix of an MMF. Our experiments show that in standard laboratory conditions, the calibration remains stable for over 90 minutes (see Supplementary, Fig. S2). Furthermore, sensitivity to bending can be mitigated using fiber modeling approach [191], or by advanced multicore-multimode fibers [192] and swept-source imaging [193], all of which are fully compatible with the proposed MMF QPI. The MMF-based noniterative QPI paves the way toward high-speed phase sensors with many applications in life science and the semiconductor industry.

APPENDIX A THEORETICAL DERIVATIONS

A.1 FOURIER TRANSFORM THEOREMS

Some theorems regarding the Fourier transform are used in the derivations in this paper; they are presented here with proof. The Fourier transform of a square-integrable function $f : \mathbb{R}^2 \rightarrow \mathbb{C}$ is written $\widehat{f} = \mathfrak{F}\{f\}$ and is defined by

$$\mathfrak{F}\{f\}(\mathbf{x}) := \int_{\mathbb{R}^2} \widehat{f}(\xi) e^{2\pi i \xi \cdot \mathbf{x}} d\xi. \quad (\text{A1})$$

Its inverse is given by

$$\mathfrak{F}^{-1}\{\widehat{f}\}(\xi) := \int_{\mathbb{R}^2} f(\mathbf{x}) e^{-2\pi i \xi \cdot \mathbf{x}} d\mathbf{x}. \quad (\text{A2})$$

Lemma .1. *For a square-integrable and sufficiently smooth function $f : \mathbb{R}^2 \rightarrow \mathbb{C}$ we have*

$$2\pi i \int \xi |\widehat{f}(\xi)|^2 d\xi = \int \overline{f(\mathbf{x})} \nabla f(\mathbf{x}) d\mathbf{x}. \quad (\text{A3})$$

Proof. Express

$$\begin{aligned} f(\mathbf{x}) &= \int \widehat{f}(\xi) e^{-2\pi i \xi \cdot \mathbf{x}} d\xi, \\ \nabla f(\mathbf{x}) &= 2\pi i \int \xi \widehat{f}(\xi) e^{-2\pi i \xi \cdot \mathbf{x}} d\xi, \end{aligned}$$

which yields

$$\int \overline{f(\mathbf{x})} \nabla f(\mathbf{x}) d\mathbf{x} = 2\pi i \int \int \int \xi' \overline{\widehat{f}(\xi)} \widehat{f}(\xi') e^{2\pi i \mathbf{x} \cdot (\xi - \xi')} d\mathbf{x} d\xi d\xi'$$

Using that $\int e^{2\pi i \mathbf{x} \cdot (\xi - \xi')} d\mathbf{x} = \delta(\xi - \xi')$ we get the desired result. ■

Theorem .2. *For a square-integrable and sufficiently smooth function $f : \mathbb{R}^2 \rightarrow \mathbb{C}$ we have*

$$\int_{\mathbb{R}^2} |f(\mathbf{x})|^2 \nabla \theta(\mathbf{x}) d\mathbf{x} = 2\pi \int_{\mathbb{R}^2} \xi |\widehat{f}(\xi)|^2 d\xi, \quad (\text{A4})$$

where $\theta = \arg(f)$ denotes the phase of f .

Proof. Express $f(\mathbf{x}) = a(\mathbf{x})e^{i\theta(\mathbf{x})}$ and apply Lemma .1:

$$\begin{aligned} 2\pi \int_{\mathbb{R}^2} \xi |\widehat{f}(\xi)|^2 d\xi &= -i \int_{\mathbb{R}^2} \overline{f(\mathbf{x})} (\nabla a(\mathbf{x}) + ia(\mathbf{x}) \nabla \theta(\mathbf{x})) e^{i\theta(\mathbf{x})} d\mathbf{x} \\ &= \int_{\mathbb{R}^2} |f(\mathbf{x})|^2 \nabla \theta(\mathbf{x}) d\mathbf{x} - i \int_{\mathbb{R}^2} a(\mathbf{x}) \nabla a(\mathbf{x}) d\mathbf{x}. \end{aligned} \quad (\text{A5})$$

The left-hand side is real-valued, as is the first term on the right-hand side. Since a is strictly real-valued, it follows that $\int a(\mathbf{x}) \nabla a(\mathbf{x}) d\mathbf{x} = 0$. ■

A.2 MAIN EQUATION

Suppose we illuminate the ultimately thin sample with various illumination patterns. We analyze corresponding fields in the Fourier plane (far field) of a lens with the focal distance F . The field of the m -th illumination pattern as it arrives at the sample plane is given by:

$$E_m^{\text{inc}} = B_m(x, y)e^{i\psi_m(x, y)}, \quad (\text{A6})$$

where $B_m(x, y)$ is the amplitude of the incident field, and $\psi_m(x, y)$ is the phase of the incident illumination, x and y are coordinates in a sample plane. The incident illumination interacts with the sample, subsequently getting the amplitude and the phase modulation. The field just after the ultimately thin sample is given by:

$$E_m^{\text{sample}}(x, y) = \gamma(x, y)E_m^{\text{inc}}(x, y) = A(x, y)B_m(x, y)e^{i(\phi(x, y) + \psi_m(x, y))}. \quad (\text{A7})$$

where $\gamma(x, y)$ is complex transmission function of the sample, $A(x, y)$ is the amplitude transmission function of the sample and $\phi(x, y)$ is the phase delay that the sample induces. According to the Fraunhofer diffraction theory, the field in the back focal plane of the thin lens is proportional to the Fourier transform of the sample field [190].

$$E_m^{\text{out}}(u, v) = \frac{e^{iC}}{\lambda F} \mathfrak{F}\{E_m^{\text{sample}}\}(u/(\lambda F), v/(\lambda F)), \quad (\text{A8})$$

where C is a coordinate-dependent phase factor, $E_m^{\text{out}}(u, v)$ is the field in the back focal plane of the lens, u, v are the coordinates in the back focal plane of the lens, λ is the wavelength of light and F is the lens focal distance.

We now apply theorem 2 with $f(\mathbf{x}) = E_m^{\text{sample}}(x, y)$ taking into account the relation stated in equation (A8) to produce a key result of this derivation, a relation between the measured centroid position (u, v) and the unknown phase gradient $\nabla\phi(x, y)$:

$$\frac{2\pi}{\lambda F} \iint \left(\frac{u}{v} \right) \cdot |E_m^{\text{out}}(u, v)|^2 du dv = \iint |E_m^{\text{sample}}(x, y)|^2 \nabla(\phi(x, y) + \psi_m(x, y)) dx dy. \quad (\text{A9})$$

We note that the left-hand side of equation (A9) corresponds to the spectral power centroid of the sample field, a measurable quantity that is given by

$$\left(\frac{u_m}{v_m} \right) = \frac{1}{P_m} \iint \left(\frac{u}{v} \right) \cdot |E_m^{\text{out}}(u, v)|^2 du dv, \quad (\text{A10})$$

where P_m is the total transmitted power expressed by the following equation:

$$\begin{aligned} P_m &= \iint |E_m^{\text{out}}(u, v)|^2 du dv = \iint |E_m^{\text{sample}}(x, y)|^2 dx dy = \\ &= \iint |E_m^{\text{inc}}(x, y)\gamma(x, y)|^2 dx dy = \iint B_m(x, y)^2 A(x, y)^2 dx dy. \end{aligned} \quad (\text{A11})$$

The equation equation (A11) links the unknown intensity transmission function of the sample $A^2(x, y)$ with the calibrated intensity distribution of incident patterns $B_m^2(x, y)$ and the measured total transmitted power P_m .

By multiplying the left and the right sides of equation (A9) by the factor $\lambda F/(2\pi P_m)$ and taking into account equation (A10) we can rewrite equation (A9) in the following form:

$$\begin{pmatrix} u_m \\ v_m \end{pmatrix} = \frac{\lambda F}{2\pi P_m} \iint B_m(x, y)^2 A(x, y)^2 \nabla(\phi(x, y) + \psi_m(x, y)) dx dy. \quad (\text{A12})$$

The left side of equation (A12) is the centroid position in the back focal plane of the lens, the integrated quantity that can be measured with the position-sensitive detector. The right-hand side of equation (A12) is a functional of the unknown phase gradient of the sample $\phi(x, y)$, the unknown intensity transmission function of the sample $A^2(x, y)$, the known (pre-calibrated) phase gradient of the illumination pattern, $\psi_m(x, y)$, the known (pre-calibrated) intensity distribution of the incident illumination $B_m^2(x, y)$ and the total optical power P_m .

APPENDIX B SPECIAL CASES

In this section, we consider equation (A12) for three special cases of illumination patterns and samples: the first case assumes complex illumination patterns and purely phase sample, the second case assumes complex illumination patterns and phase and amplitude sample and the third case assumes binary amplitude modulation of illumination patterns and purely phase sample.

B.1 COMPLEX ILLUMINATION OF A PHASE-ONLY SAMPLE

If the illumination patterns $S_m(x, y)$ are complex-valued, and the sample does not induce amplitude modulation of the incident patterns $A(x, y) \equiv 1$ for $\forall x, y$, we can write:

$$\begin{pmatrix} u_m \\ v_m \end{pmatrix} = \frac{\lambda F}{2\pi P_m} \iint B_m(x, y)^2 \nabla(\phi(x, y) + \psi_m(x, y)) dx dy. \quad (\text{B13})$$

We can recognize that the very last term of the right part of the equation is the spectral power centroid position in the absence of a sample:

$$\begin{pmatrix} u_{m0} \\ v_{m0} \end{pmatrix} = \frac{\lambda F}{2\pi P_m} \iint B_m(x, y)^2 \nabla\psi_m(x, y) dx dy. \quad (\text{B14})$$

Therefore, we can rewrite equation (B13):

$$\begin{pmatrix} \Delta u_m \\ \Delta v_m \end{pmatrix} = \begin{pmatrix} u_m - u_{m0} \\ v_m - v_{m0} \end{pmatrix} = \frac{\lambda F}{2\pi P_m} \iint B_m(x, y)^2 \nabla\phi(x, y) dx dy. \quad (\text{B15})$$

where $(\Delta u_m, \Delta v_m)$ is the shift of the power centroid position induced by the phase modulation of the sample. The introduction of $\begin{pmatrix} u_{m0} \\ v_{m0} \end{pmatrix}$ allows to eliminate the necessity of the complicated interferometric measurements of the phase of the incident patterns $\psi_m(x, y)$.

B.2 COMPLEX ILLUMINATION OF A PHASE AND AMPLITUDE SAMPLE

Here we consider the case where the sample is not a purely phase object but modulates both the amplitude and phase of the incident radiation. To extract the spatial phase profile of

the sample according to equation (A12), we need to know the phase and intensity profiles of the incident patterns. As the amplitude modulation of the sample, $A(x, y)$, can be calculated from the total transmitted power measurements P_m and the illumination patterns can be fully characterized via pre-calibration, we can define the functionals α and β as:

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{\lambda F}{2\pi P_m} \iint (A(x, y) B_m(x, y))^2 \nabla \psi_m(x, y) dx dy. \quad (\text{B16})$$

and rewrite equation (A12) as:

$$\begin{pmatrix} u_m - \alpha \\ v_m - \beta \end{pmatrix} = \frac{\lambda F}{2\pi P_m} \iint (A(x, y) B_m(x, y))^2 \nabla \phi(x, y) dx dy, \quad (\text{B17})$$

We are interested not only in the phase map of the sample but in the phase map changes over time. Assuming that the amplitude transmission function of the sample does not change we can write down the set of equations similarly to pure phase sample case equation (B15). The calculation of the sample phase map does not require measuring phase distributions of the illumination patterns:

$$\begin{pmatrix} u_{m0} \\ v_{m0} \end{pmatrix} = \frac{\lambda F}{2\pi P_m} \iint (A(x, y) B_m(x, y))^2 \nabla (\psi_m(x, y) + \phi_{t1}(x, y)) dx dy. \quad (\text{B18})$$

$$\begin{pmatrix} \Delta u_m \\ \Delta v_m \end{pmatrix} = \begin{pmatrix} u_m - u_{m0} \\ v_m - v_{m0} \end{pmatrix} = \frac{\lambda F}{2\pi P_m} \iint B_m(x, y)^2 A(x, y)^2 \nabla \phi_{t2}(x, y) dx dy. \quad (\text{B19})$$

where $(\Delta u_m, \Delta v_m)$ is the shift of the power centroid position induced by the phase difference at the different moments in time.

B.3 BINARY ILLUMINATION OF A PHASE-ONLY SAMPLE

To compare the equations derived in the current manuscript with the equations reported previously in the literature in reference [176], we also rewrite equation (A12) for the simplified case of ideal binary patterns and a pure phase sample. Taking into account that in this case the incident illumination patterns have planar wavefronts $\nabla \psi_m(x, y) \equiv 0$, the sample does not introduce amplitude modulation $A(x, y) \equiv 1$, and the patterns are binary $B_m^2(x, y) = B_m(x, y) = 1$ for $\forall x, y \in \mathcal{A}_m$ and $B_m^2(x, y) = B_m(x, y) = 0$ for $\forall x, y \notin \mathcal{A}_m$, we get:

$$\begin{pmatrix} u_m \\ v_m \end{pmatrix} = \frac{\lambda F}{2\pi P_m} \iint_{\mathcal{A}_m} \nabla \phi(x, y) dx dy, \quad (\text{B20})$$

where $\mathcal{A}_m$ is the region of the sample where the m th illumination pattern is non-zero, and $P_m = \iint_{\mathcal{A}_m} 1 dx dy$ is the area corresponding to that region. This result for the ideal simplified case is identical to that obtained in [176].

APPENDIX C EQUATIONS IN A MATRIX FORM

Here we discuss the proposed procedure to solve the equations and reconstruct the unknown phase of the sample by using the discretized matrix form. We assume that the

image plane and Fourier plane fields are sampled on a $N_1 \times N_2$ uniform grid with $N = N_1 \cdot N_2$ grid points, where (u', v') , (x', y') are the grid coordinates in the Fourier and the sample plane, respectively, and $\epsilon \times \epsilon$ is the pixel size in the sample plane. The discrete versions of the incident illumination and the sample can be introduced as the values averaged over a respective pixel area $B_m[x', y'] \exp(\psi_m[x', y'])$ and $A[x', y'] \exp(\phi[x', y'])$. Using these notations and substituting continuous integrals with discrete sums we can transform the derived integral equations to the systems of linear equations.

C.1 GENERAL CASE

Firstly, we derive the equation to reconstruct the amplitude transmission function of the sample $A[x', y']$ from the total transmitted power P_m . We can write the discretized version of equation (A11) as:

$$P_m = \sum_{x'} \sum_{y'} S_m[x', y'] A[x', y']^2 \epsilon^2, \quad (\text{C21})$$

where $S_m[x', y'] = B_m^2[x', y']$ is the discretized power distribution of incident speckle patterns.

We introduce vector $\mathbf{p} \in \mathbb{R}^M$ representing the total transmitted power for each speckle illumination pattern with P_m being the m^{th} element, the matrix $\mathbf{S} \in \mathbb{R}^{M \times N}$ containing the power distributions of the speckle patterns, and the vector $\mathbf{a} \in \mathbb{R}^N$ representing the intensity transmission function of the sample with elements $A[x_n, y_n]^2$. Using these notations, we rewrite the set of equations C21 in the matrix form:

$$\mathbf{p} = \mathbf{S}\mathbf{a}, \quad (\text{C22})$$

and its solution using the Moore-Penrose pseudoinverse:

$$\mathbf{a} = \mathbf{S}^\dagger \mathbf{p}, \quad (\text{C23})$$

getting the first equation of the set of equations (3) of the main manuscript. The amplitude transmission $A[x', y']$ is reconstructed by the transformation of the vector $\mathbf{a}$ into the 2D map $A^2[x', y']$ and taking the square root. We note that the solution to equation (C22) may not exist (e.g., when $M > N$) nor be unique (e.g., when $M < N$ as often in experiments). The pseudo-inverse (cf. C23) provides a minimum-norm least-squares solution in these cases. In particular, in experimentally relevant settings we often have $M < N$ and we retrieve $\mathbf{a}$ up to components in the null-space of $\mathbf{S}$. We can explicitly compute the pseudo-inverse through the singular value decomposition (SVD), and regularise the computation by truncating small singular values.

Secondly, we derive the equation to reconstruct the phase map of the sample $\phi[x', y']$. We use the discretized version of equation (B17)

$$\begin{pmatrix} u_m - \alpha_m \\ v_m - \beta_m \end{pmatrix} = \frac{\lambda F}{2\pi P_m} \sum_{x'} \sum_{y'} S_m[x', y'] (A[x', y'])^2 \nabla \phi[x', y'] \epsilon^2. \quad (\text{C24})$$

Introducing the vectors $\mathbf{u} \in \mathbb{R}^M$ and $\mathbf{v} \in \mathbb{R}^M$ representing the x - and y -coordinates of the centroid in the Fourier plane, vectors $\boldsymbol{\alpha} \in \mathbb{R}^M$ and $\boldsymbol{\beta} \in \mathbb{R}^M$ representing the functionals of

the incident speckle patterns and ϕ_x, ϕ_y denoting the derivatives in the x and y directions of ϕ on the grid, equation (C24) can be written as:

$$\begin{pmatrix} \mathbf{u} - \boldsymbol{\alpha} \\ \mathbf{v} - \boldsymbol{\beta} \end{pmatrix} = \begin{pmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{pmatrix} \begin{pmatrix} \phi_x \\ \phi_y \end{pmatrix}, \quad (\text{C25})$$

with

$$\mathbf{M} = \frac{\lambda F}{2\pi} \text{diag}(\mathbf{p})^{-1} \mathbf{S} \text{diag}(\mathbf{a}). \quad (\text{C26})$$

With this, the unknown quantities ϕ_x, ϕ_y can be calculated by applying $\mathbf{M}^\dagger$ to $\mathbf{u} - \boldsymbol{\alpha}$ and $\mathbf{v} - \boldsymbol{\beta}$. The phase ϕ can be computed from ϕ_x, ϕ_y by numerical integration.

C.2 DIFFERENTIAL SCHEME

Similarly, we write the matrix form of equation (B18) as:

$$\begin{pmatrix} \Delta \mathbf{u} \\ \Delta \mathbf{v} \end{pmatrix} = \begin{pmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{pmatrix} \begin{pmatrix} \phi_x \\ \phi_y \end{pmatrix}, \quad (\text{C27})$$

where $\Delta \mathbf{u}$ and $\Delta \mathbf{v}$ are vectors with the elements $\Delta u_m = u_m - u_{m0}$ and $\Delta v_m = v_m - v_{m0}$ representing the relative shift of the centroid position in respect to the reference point in x - and y -directions, respectively. In this case, the unknown phase gradients can be calculated as:

$$\begin{pmatrix} \phi_x \\ \phi_y \end{pmatrix} = \begin{pmatrix} \mathbf{M}^\dagger & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^\dagger \end{pmatrix} \begin{pmatrix} \Delta \mathbf{u} \\ \Delta \mathbf{v} \end{pmatrix} \quad (\text{C28})$$

which is the second equation in the set of equations (3) of the main manuscript.

QUANTITATIVE PHASE IMAGING WITH A MULTIMODE FIBER: SUPPLEMENTAL DOCUMENT

1 EXPERIMENTAL SETUP AND ALIGNMENT

The detailed image of the experimental setup is presented in Fig. S1 and stability measurements are presented in Fig. S2.

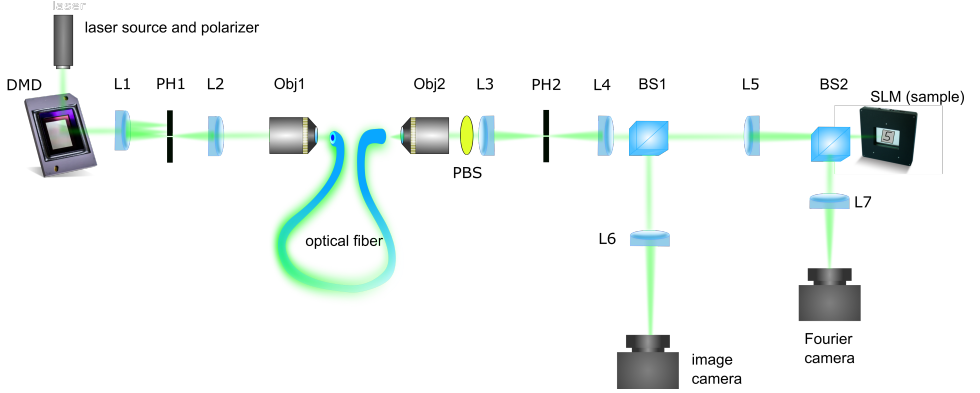


Figure S1: Detailed schema of the experimental setup. By a 4f system consisting of lenses L1, L2 (Thorlabs, AC508 – 150-A-ML $\varnothing$ 2) the DMD screen is imaged on the back focal plane of Obj1. Pinhole PH1 (Thorlabs, SM1D12C) only allows the +1st diffraction order of the DMD to pass. The output illumination from the MMF is horizontally polarized by PBS1 (Thorlabs, PBS251 - 1") and projected onto a 500 μm square pinhole PH2 (Thorlabs, S500QK - $\varnothing$ 1) by a microscope objective Obj2, (Leica, HC-PL-FLUOTAR, 40 $\times$, NA = 0.6, f_{Obj2} = 5mm) and a lens L3, (Thorlabs, AC254 – 200-A-ML) with the overall magnification of 40 $\times$. Subsequently, the pinhole is imaged on the surface of the liquid crystal SLM (Meadowlark Optics, E19x12-350-850-HDM8, 60Hz frame rate) by L4 (Thorlabs, AC254 – 080-A-ML) and L5 (Thorlabs, AC254 – 125-A-ML). We experimentally measure 63 $\times$ magnification of a fiber core facet ($\varnothing$ = 50 μm) onto the SLM plane, which therefore confirms the expected magnification. The SLM is imaged onto the 2.4 μm -pixel-size image camera (IDS, U3 – 3070C P1/1.8Monochrome U, 118 fps) using L5 and L6 (Thorlabs, AC254 – 125-A-ML) without magnification. The Fourier plane of the SLM is imaged onto the Fourier camera (IDS, U3 – 3880C P1/1.8Monochrome U, 58 fps) with a Fourier lens, L7 (Thorlabs, AC254 – 200-A-ML). DMD, digital micromirror device; SLM, liquid crystal spatial light modulator; BS, beam splitter; PBS, polarization beam splitter; L, lens; PH, pinhole; Obj, objective

To ensure the accuracy of optical alignment and data processing, we conducted two sets of calibration measurements using simple linear phase gradient samples. In the first set of experiments, we projected various linear phase ramp samples onto the SLM and measured the resulting phase shifts. For a linear phase ramp along the x -axis, the shift in the Fourier plane depends solely on the phase gradient and is independent of the illumination pattern. This can be immediately seen from Eq. C26:

$$\begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix} = \begin{pmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{pmatrix} \begin{pmatrix} \phi_x \\ \phi_y \end{pmatrix}, \quad (\text{S1})$$

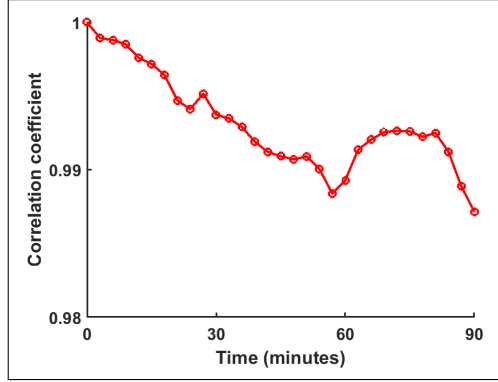


Figure S2: Long-term stability of the experimental system. A focused beam was projected onto the input facet of a multimode fiber, and the resulting speckle pattern was recorded on a camera every 3 minutes over 90 minutes. The correlation coefficient between the first image (at $t = 0$) and each subsequent image was computed using the normalized cross-correlation formula.

Taking into account that

$$\nabla\phi_x = \mathcal{K} \cdot \frac{\Delta\phi_x}{N_1\epsilon}, \quad (\text{S2})$$

$$\nabla\phi_y = \mathcal{K}, \quad (\text{S3})$$

$$\text{diag}(\mathbf{A}) = \mathbf{I}, \quad (\text{S4})$$

where linear vector $\nabla\phi_x$ ($N \times 1$) is the flattened x component of the phase gradient, linear vector $\nabla\phi_y$ ($N \times 1$) is the flattened y -component of the phase gradient, $\mathcal{K}, \mathcal{K}$ are the linear vectors ($N \times 1$) consisting of ones and zeros respectively, $\mathbf{I}$ is a unity matrix ($M \times M$), N_1 is the number of pixels in the field of view along x -direction, ϵ is the pixel size, N is the total number of pixels. Equation 3 from the main manuscript is simplified to:

$$\Delta\phi_x = \frac{2\pi N_1\epsilon}{\lambda F}(u - u_0), \quad (\text{S5})$$

$$\Delta\phi_y = 0, \quad (\text{S6})$$

where $\Delta\phi_x$ is the phase ramp in radians along x -axis, $\Delta\phi_y$ is the phase ramp along y -axis and $N_1\epsilon = 780 \mu\text{m}$ is the size of the field of view in x -direction.

We performed the measurements for various patterns projected onto the SLM with phase ramps $\Delta\phi_x^{SLM} = 6\pi, 11\pi, 16\pi, 20\pi$, and 30π . We compared the theoretical and experimental phase ramp values and plotted $\Delta\phi_x^{SLM}$ and $\Delta\phi_x$ as functions of the measured shift in the Fourier plane $u - u_0$. The results are presented in Figure S3(a). The excellent agreement between the theoretical and experimental phase ramps indicates a well-aligned optical system and the correct calibration of the SLM.

In the second set of calibration measurements, we used the proposed approach to image a simple linear phase gradient, $\Delta\phi_x^{SLM} = 16\pi$. A comparison between the imaging result and the SLM pattern in Figure S3(b,c) reveals an excellent agreement, confirming the functionality of our experimental setup. The success of the calibration measurements has enabled us to

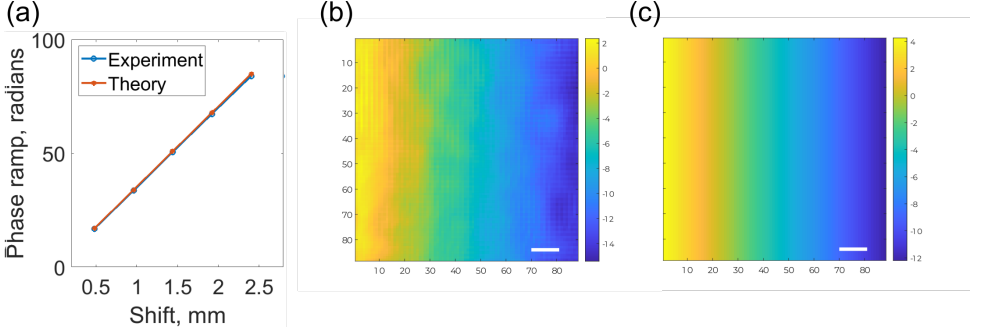


Figure S3: (a) Theoretical $\Delta\phi_x^{SLM}$ (solid blue line) and experimentally measured phase ramps $\Delta\phi_x$ (solid orange line) as a function of the measured shift $u - u_0$ in the Fourier plane. (b) The imaging result for the linear phase ramp sample in comparison with the SLM pattern (c). The applied phase ramp is $\Delta\phi_x^{theory} = 16\pi$.

shift our focus towards imaging more complex targets, and the results are presented in the main manuscript.

2 DIFFRACTION LIMIT OF THE OPTICAL SYSTEM

To accurately provide the simulations of the proposed approach, we first determine the cutoff frequency of our experimental optical system. In Fig. S4(a) we present the simulated intensity distribution of an ideal speckle pattern [194] and in Fig. S4(b) the experimentally measured on our sample. A mean power spectrum of speckle patterns can be calculated as:

$$PS(f'_x, f'_y) = \langle \mathfrak{F}\{I_i(x, y)\}^2 \rangle, \quad (S7)$$

where $f'_x = f_x/f_0$, $f'_y = f_y/f_0$ are the normalized spatial frequencies, $f_0 = 2NA/\lambda$ is the theoretical cutoff frequency, $NA = 0.0035$, $\lambda = 640$ nm and $\langle \cdot \rangle$ denotes the ensemble averaging.

The averaged power spectra in logarithmic scale are presented in Fig. S4 (c,d) for an ideal and measured speckle patterns respectively. White dashed circles mark the spatial frequency f_0 . The horizontal and vertical cross-sections of the ideal and experimentally measured power spectrum are presented in Fig. S4 (e,g and f,h), respectively. Our experimental results match the theoretical expectation and numerical simulation: power spectrum dramatically decays at frequencies higher than f_0 . The experimental power spectrum has a more complicated structure compared to the ideal speckles. Four diffraction orders observed along f_y axis in Figure S4(d) are attributed to the interference pattern and pixel structure of spatial light modulator surface superimposed with the experimental speckle patterns. The periodic modulations with two characteristic frequencies result in 4 peaks, which are not part of the true experimental power spectrum.

The experimentally measured power spectrum is asymmetric. It is steeper along f_y axis compared to the f_x axis. Most of the experimental speckles are slightly elongated along y-axis that can be also observed in Fig. S4(b). This can be explained by a speckle structure generated within a round-core multimode fiber. We experimentally cut the area close to the core edge. To summarize, the performed analysis demonstrates that the experimental illumination speckle patterns are diffraction-limited with the cutoff frequency f_0 .

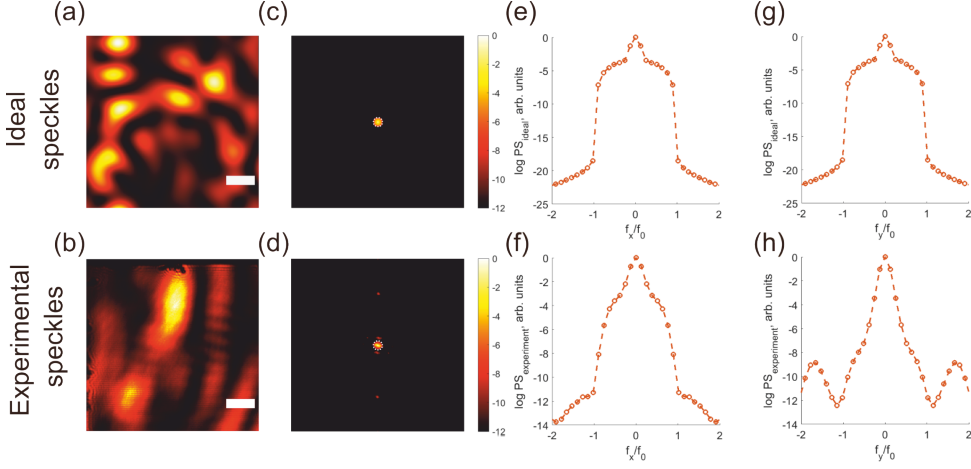


Figure S4: (a,b) Examples of intensity distributions of an ideal (a) and experimentally measured (b) speckle patterns. The scale bar is equal to the Abbe diffraction limit of $95 \mu\text{m}$. (c,d) Averaged power spectra of ideal (c) and experimentally measured (d) speckle patterns in logarithmic scale. The white dashed circles mark the cutoff frequency f_0 . (e-h) Horizontal (e) and vertical (g) cross-sections of (c) and horizontal (f) and vertical (h) cross-sections of (d).

Knowing the cutoff frequency of our optical system, we can simulate the proposed approach and estimate the theoretical resolution of fiber-based quantitative phase imaging. To define the spatial resolution of our approach, we simulate imaging of a sample $\gamma(x, y)$ consisting of two phase-only objects with the same Gaussian-like phase distribution ($\text{FWHM} = 0.4\lambda/NA$) separated by different distances: λ/NA , $0.76\lambda/NA$ (approximately corresponding to coherent diffraction limit), and $0.5\lambda/NA$.

Firstly, we simulated conventional coherent diffraction-limited imaging. The field in the image plane of a coherent diffraction-limited optical system $E_{dl}(u, v)$ is defined as [190]:

$$E_{dl}(u, v) = \mathfrak{F}^{-1}\{H_{coh}(f_x, f_y) \cdot \mathfrak{F}\{\gamma(x, y)\}\} \quad (\text{S8})$$

where $H_{coh}(f_x, f_y)$ is a low-pass filter with the coherent cutoff frequency $f_{coh} = f_0/2$, f_x, f_y are the spatial frequencies, x, y are the coordinates in the sample plane, and u, v are the coordinates in the image plane. The diffraction-limited phase map is $\phi_{dl}(u, v) = \arg(E_{dl})$. The results of the simulations are presented in Figure 5.2(b,g,l) of the main manuscript.

As the next step, we simulate the proposed fiber-based imaging. We generated 8464 ideal speckle patterns. The characteristic intensity and phase distributions are presented in Figure S5(a,b), respectively. We performed the simulation of fiber-based QPI with the speckle patterns following the procedure described in the main text. The pseudoinverse was calculated using the MATLAB function `pinv`. The calculated gradients for one of the samples are presented in Figure S5(e,g) together with the ground truth gradients presented in Figure S5(f,h) for x - and y - directions, respectively. The phase map was calculated from partial derivatives using the MATLAB script [195]. The reconstruction result together with the ground truth is presented in Figure S5(c,d). The results of the simulations for all samples are presented in Figure 5.2(c,h,m) of the main manuscript.

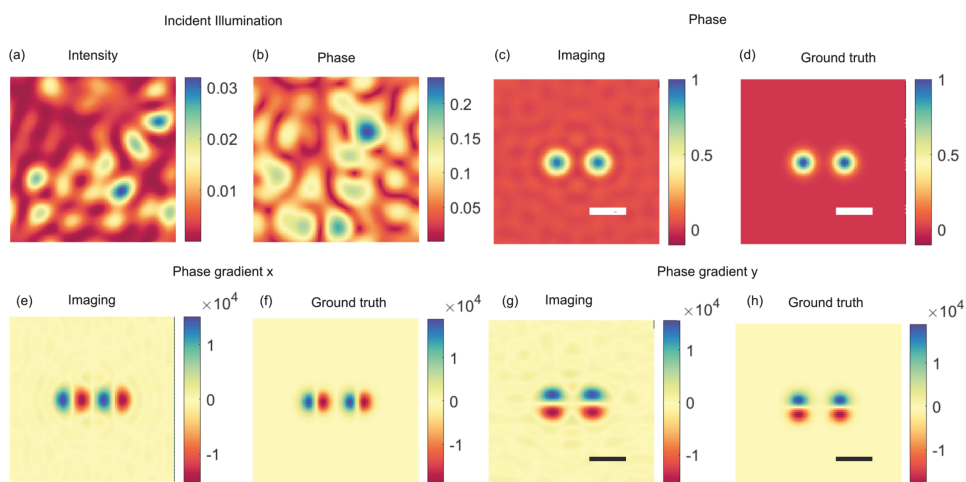


Figure S5: (a,b) Characteristic intensity and phase distributions, respectively. (c,d) Reconstructed and ground truth sample phase map. (e,f) Reconstructed and ground truth gradient in x-direction. (g,h) Reconstructed and ground truth gradient in y-direction. The scalebar corresponds to the coherent diffraction limit $140 \mu\text{m}$.

6

SUMMARY AND OUTLOOK

SUMMARY

This thesis investigates how the complex speckle patterns generated by multimode optical fibers can be understood, controlled, and exploited for illumination and imaging applications.

Firstly, **in the Chapter 2**, a numerical framework is introduced to simulate light propagation in MMFs. A mode solver operates in both scalar and vector regimes, allowing polarization effects to be included. The mode-solver allows us to study the systematic tuning of wavelength, input beam position, and beam size, and produces speckle patterns. Using this framework, speckle statistics is connected to fiber parameters, providing physical insight into MMFs light propagation and its dependence on experimental conditions.

In Chapter 3, focuses on experimental ways for measuring the complex-valued TM of MMF, with particular emphasis on phase retrieval methods: in-line and off-axis holography. The TM is obtained in a spatial basis by scanning the input facet of MMF and reconstructing the complex optical field. Two interferometric phase-retrieval strategies off-axis and in-line holography are implemented within the same experimental setup and systematically compared. Diffraction-limited focal spots and arbitrary intensity distributions are generated at the fiber output, with performance quantified in terms of power ratio and correlation with target patterns. The analysis reveals a fundamental trade-off between the two approaches. Off-axis holography, using an external plane-wave reference, enables accurate reconstruction of complex free-form illumination patterns with high spatial fidelity, while in-line holography benefits from a common-path geometry that provides superior stability for diffraction-limited focusing. However, the use of an internally generated speckled reference in the in-line configuration introduces phase singularities and spatially varying phase offsets, which limit its ability to reproduce extended or complex targets. These results establish clear guidelines for selecting phase-retrieval methods depending on whether the application prioritizes single-spot focusing, or projection of complex illumination patterns.

Studying the wavelength dependence of an MMF speckle pattern is important not only for speckle suppression but also for using speckles as an illumination basis. **In Chapter 4**, wavelength scanning of the input fiber facet generates a sequence of distinct and reproducible speckle patterns at the fiber output. These patterns serve as structured illuminations, and the sample images are reconstructed using compressive sensing algorithm. The imaging

performance is systematically analyzed as a function of the spectral bandwidth and image size. A key outcome of this chapter is the robust demonstration of a compact single-mode-MMF probe, in which a single-mode fiber delivers spectrally tunable illumination into an MMF that produces speckles. This setup combines efficient light delivery with wavelength-tuned speckle generation, resulting in a thin, flexible, and robust imaging probe.

Spatial scanning of the MMF input facet was used in **Chapter 5** to create independent speckle patterns for quantitative phase imaging (QPI). These speckle fields are employed to probe the sample. A new non-interferometric phase imaging method is demonstrated, in which phase information is reconstructed from intensity-only measurements. The reconstruction exploits the linear relationship between speckle-induced wavefront gradients and measured centroid shifts, enabling direct, non-iterative phase recovery. The achievable spatial resolution is set by the speckle grain size and is not limited by interferometric constraints. This approach enables QPI through an MMF using a compact optical setup and complex random illumination. The results highlight the potential of complex speckle-based illumination schemes for phase microscopy.

OUTLOOK

The results presented in this thesis demonstrate that multimode fiber speckle patterns can be understood and exploited as a powerful resource for illumination engineering, imaging, and optical metrology. The results discussed in summary suggest several directions for future research.

In this work, only a single input polarization state was controlled, with phase modulation, we can control the output field. Spatial coherence was reduced using random methods, including spectral bandwidth scanning and spatial scanning of the MMF input, combined with temporal averaging of independent speckle realizations on the camera. This provided effective coherence reduction without requiring precise amplitude or polarization control. Future improvements could be achieved by implementing full-field modulation, including control of both amplitude and phase, as well as both orthogonal polarization components. This would increase the number of controllable degrees of freedom and improve projection fidelity. Replacing the current random coherence-control approaches with deterministic methods. Such control would enable illumination schemes optimized for specific imaging and metrology tasks, where partial coherence and arbitrary illumination shape are often required to suppress interference artifacts while preserving phase sensitivity.

An important future direction is the extension of broadband illumination approaches toward broadband single-shot imaging. Instead of generating spectrally diverse speckle patterns sequentially, broadband illumination could be used to encode multiple spectral realizations simultaneously. After a wavelength-resolved calibration of the fiber response, all spectral components can be recombined and used in a single measurement, enabling compressive sensing without wavelength scanning during acquisition. Such an approach would significantly reduce measurement time and data volume.

Finally, the QPI approach demonstrated here should be extended to more realistic experimental scenarios. Testing the method on biological or industrial phase samples, as well as exploring alternative scattering media with improved mechanical stability, would provide valuable insight into practical performance limits. In addition, the use of more advanced reconstruction algorithms, including regularization or learning-based approaches,

may further reduce noise and improve phase reconstruction quality.

Future developments along the directions outlined above are expected to further expand their applicability in optical metrology, microscopy, and minimally invasive sensing.

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LIST OF PUBLICATIONS

JOURNAL ARTICLES

The research presented in this thesis has resulted in the following publications.

1. Ivanina A, Lochocki B, Koglbauer A, Sokolov S, Goorden SA, Amitonova LV: *Transmission matrix of a multimode fiber: In-line vs off-axis holography*. PLoS One 21(2): e0340823, 2026. DOI: <https://doi.org/10.1371/journal.pone.0340823>

Author contribution: The candidate designed the experimental framework, performed the simulations and experiments, interpreted the results, and prepared the manuscript, including subsequent revisions.

2. Lochocki B, Ivanina A, Bandhoe A, de Boer JF, Amitonova LV: *Swept-source multimode fiber imaging*. Scientific Reports 13, 8071, 2023. DOI: <https://doi.org/10.1038/s41598-023-34062-6>

Author contribution: The candidate contributed to the experimental work, data analysis, and manuscript preparation.

3. Ivanina A, Marshall M, Abrashitova K, van Leeuwen T, Amitonova LV: *Quantitative phase imaging with a multimode fiber*. APL Photonics 10, 046116, 2025. DOI: <https://doi.org/10.1063/5.0253648>

Author contribution: The candidate carried out the experimental implementation and contributed to drafting and revising the manuscript.

ARTICLES IN THE PREPARATION

1. Ivanina A, Lochocki B, Amitonova LV *Top-hat Illumination Through Multimode Fibers with Different Core Geometries*.

Author contribution: The candidate carried out the experimental implementation, performed the simulations, and interpreted the results.

2. Ivanina A, Biryg S, Goorden SA, Koglgauer A, Amitonova LV *Illumination Engineering for Optical Metrology*.

Author contribution: The candidate conducted the initial experimental work, designed the setup, and interpreted the results.

CONFERENCE PARTICIPATION

1. *Speckle mitigation by multimode fibers*,
Ivanina A, Lyu Z, Lochocki B, Amitonova LV,
Advanced Photonics Congress, Maastricht, The Netherlands, 2022.
Poster presentation
2. *Measuring the transmission matrix of a multimode fiber: on-axis vs off-axis holography*,
Ivanina A, Lochocki B, Amitonova LV,
SPIE Optics + Optoelectronics, Prague, Czech Republic, 2023.
Oral presentation
3. *Wavelength Scanning Multimode Fiber Imaging*,
Lochocki B, Ivanina A, Bandhoe A, de Boer JF, Amitonova LV,
CLEO/Europe-EQEC, Munich, Germany, 2023.
Poster presentation
4. *Self-reference transmission matrix measurements of MMF*,
Ivanina A, Amitonova LV,
SPIE Photonics Europe, Strasbourg, France, 2024.
Oral presentation
5. *Complex speckle illumination for quantitative phase imaging*,
Ivanina A, Marshall M, Abrashitova K, van Leeuwen T, Amitonova LV,
CLEO/Europe-EQEC, Munich, Germany, 2025.
Oral presentation

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These four years have passed by incredibly fast. My journey started in December 2021 when I arrived in the Netherlands during the last COVID lockdowns. Moving to another country was both an exciting and challenging experience. I left the not-so-sunny Saint Petersburg for the much sunnier Amsterdam (yesyes).

It was a unique way to start a new chapter of my life, moving to a new country and spending the first month in quarantine. A few months later, everything seemed to be improving, and life was about to settle down. But then in February 2022, the war started. The excitement of starting a new life abroad turned into fear, and I felt disoriented and unsure about where I was and why I was there. The future seemed uncertain. Over time, I have truly come to love the place where I am and the opportunities that have opened up for me. During these past four years, I have rebuilt and rediscovered myself on several occasions. I am deeply grateful for this experience and for the significant professional and personal growth that it has brought me.

For more in-depth reflections, I will turn to my personal journal, but here, I would like to express my gratitude to all those who have been part of this incredible PhD journey, those who have supported me and helped me understand my strengths and weaknesses.

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Sasha, June 2026

